Quad Meshing

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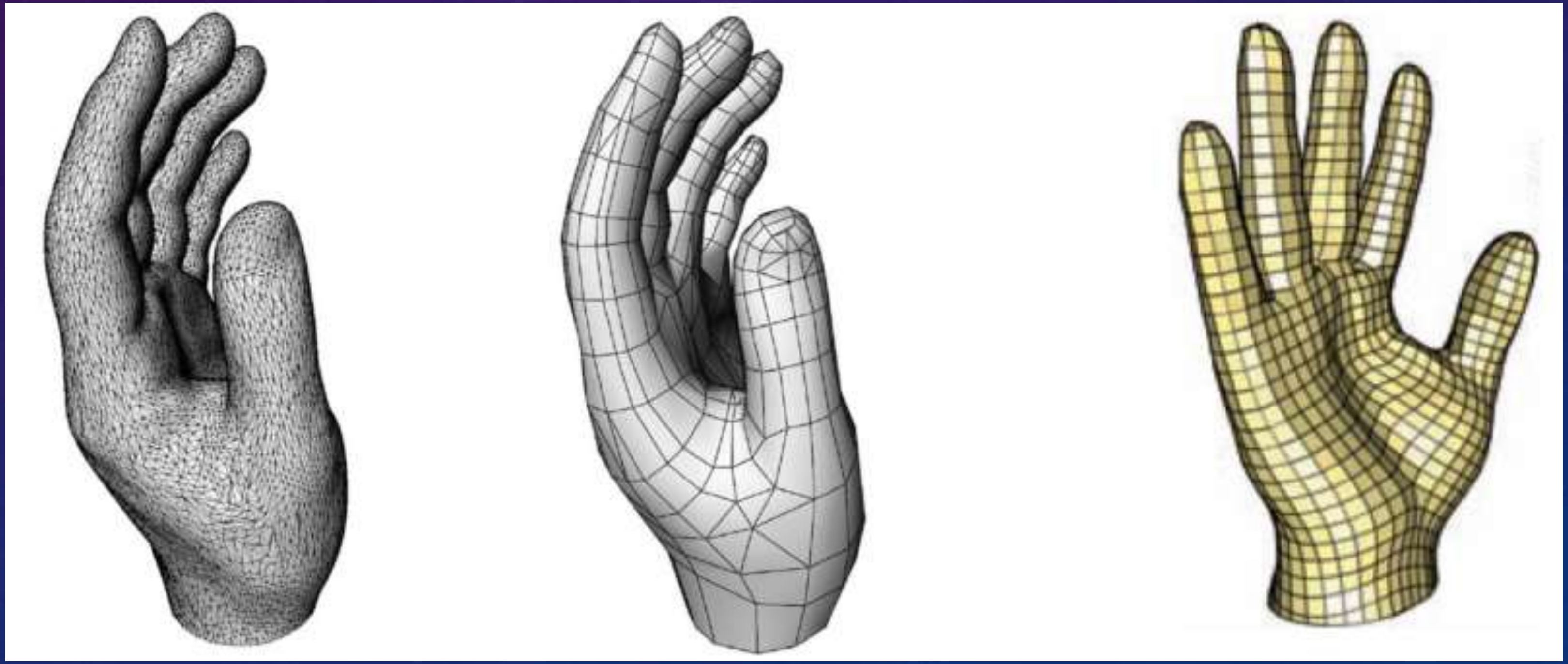
https://qingfang1208.github.io/

Quad Meshing

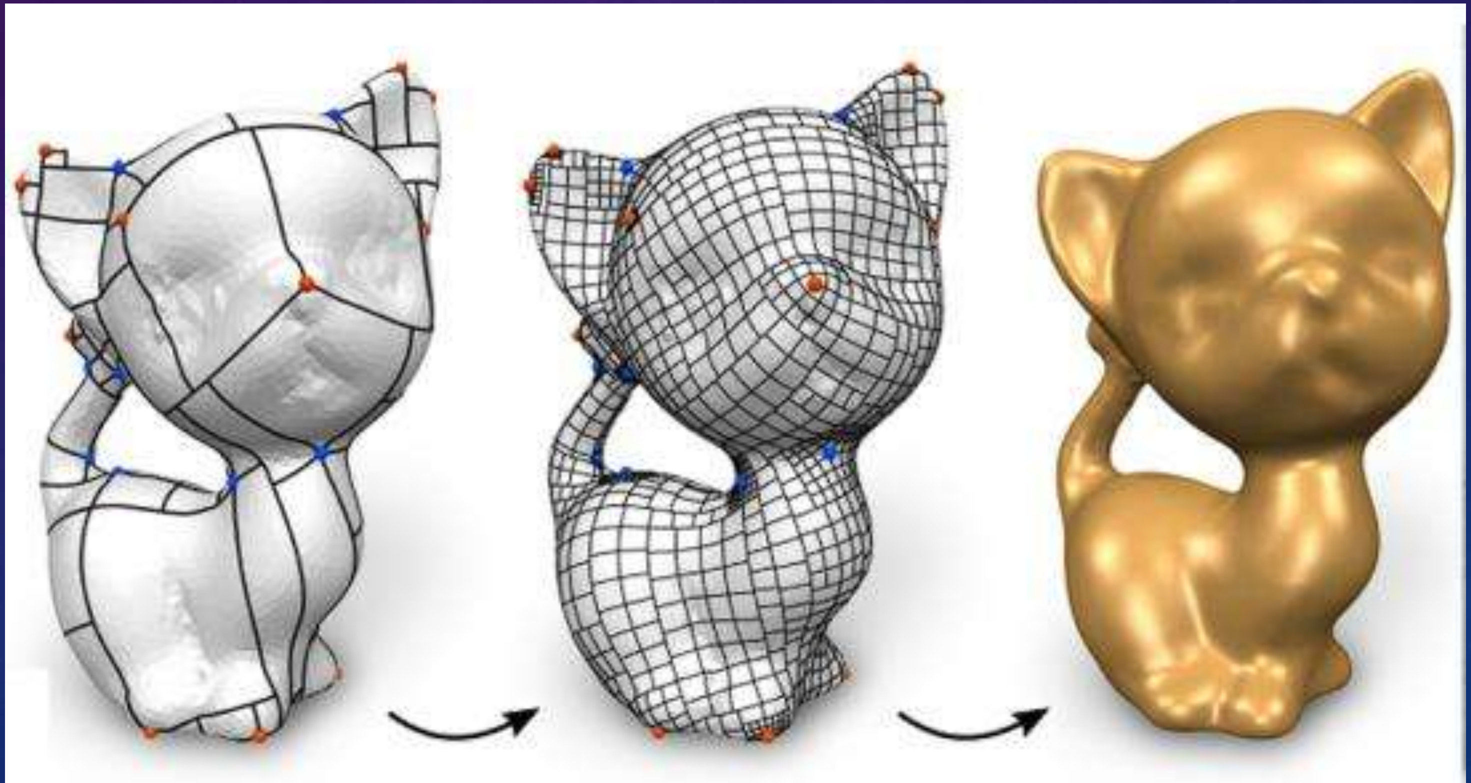
- Generate a quad (or quad-dominant) mesh that approximates the input



Input　　Quad Dominant Mesh　　Quad Mesh

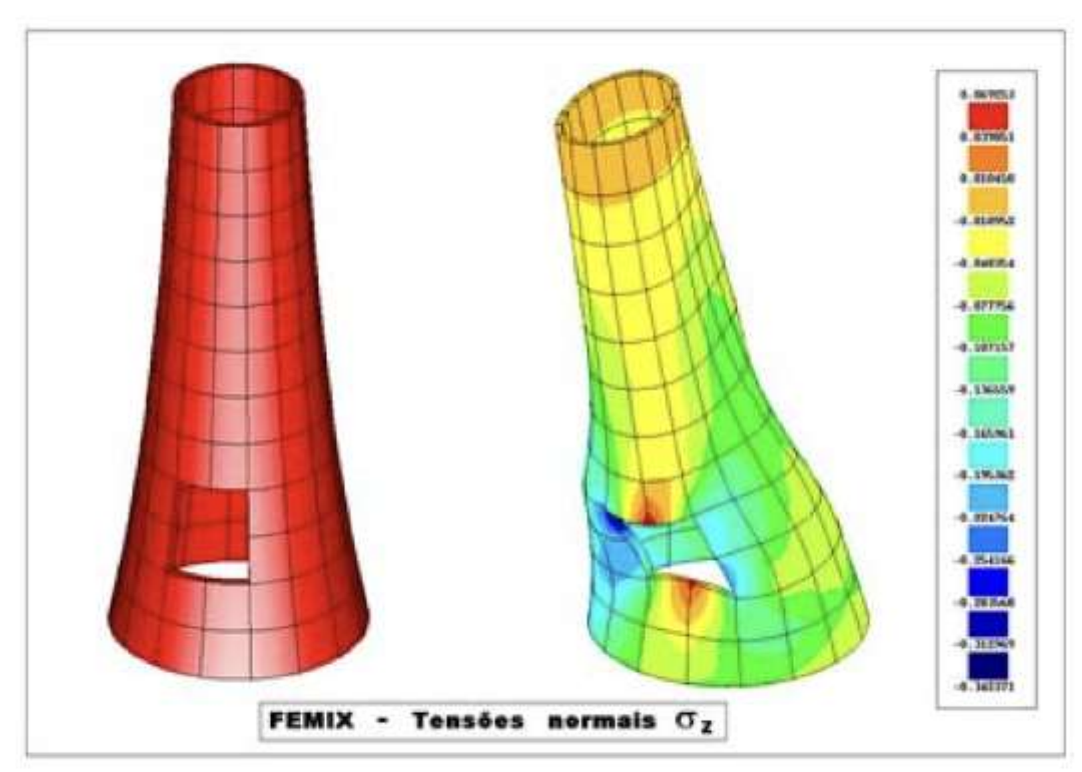
Applications

- Fitting B-spline surfaces



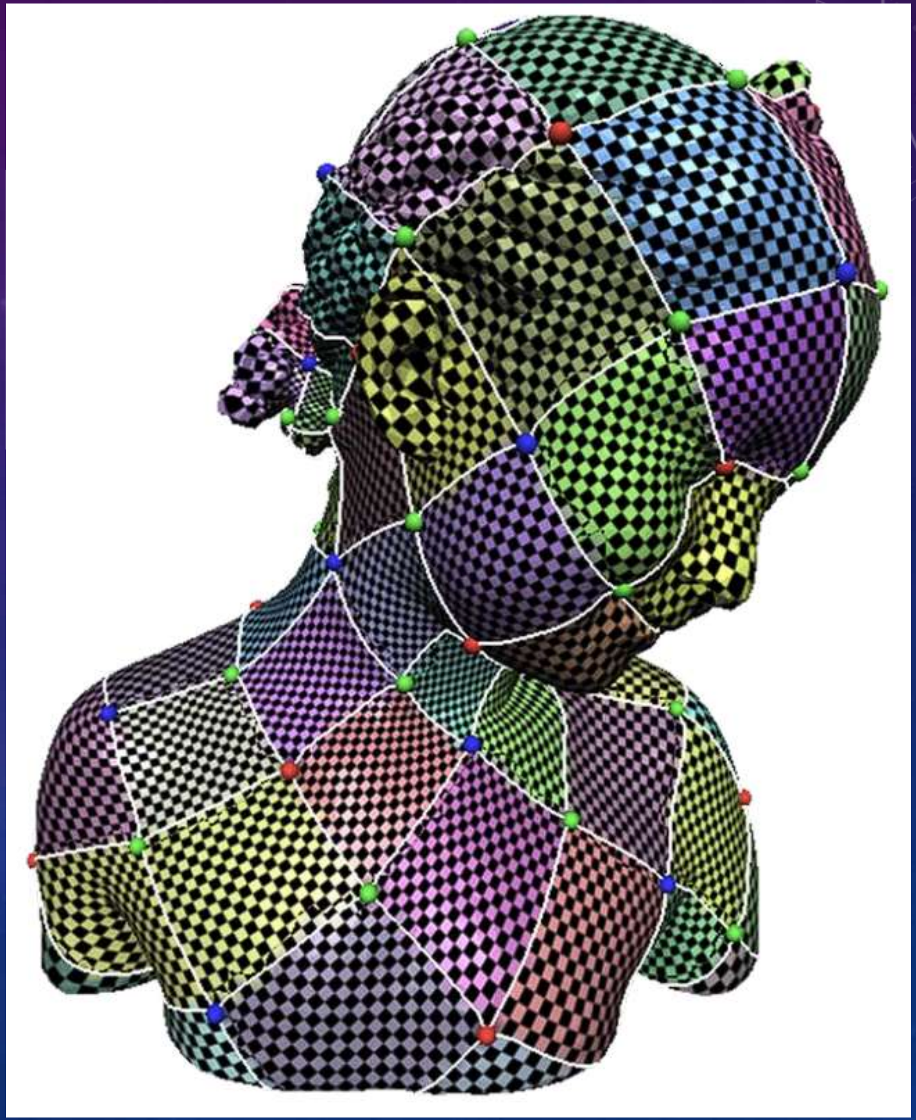
Applications

- Fitting B-spline surfaces
- Simulation



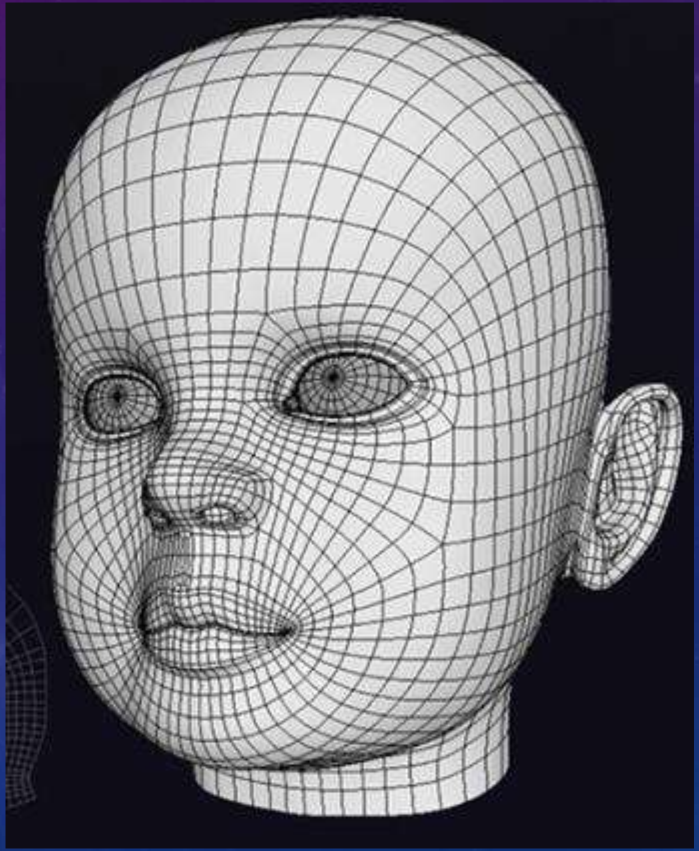
Applications

- Fitting B-spline surfaces
- Simulation
- Texture atlas



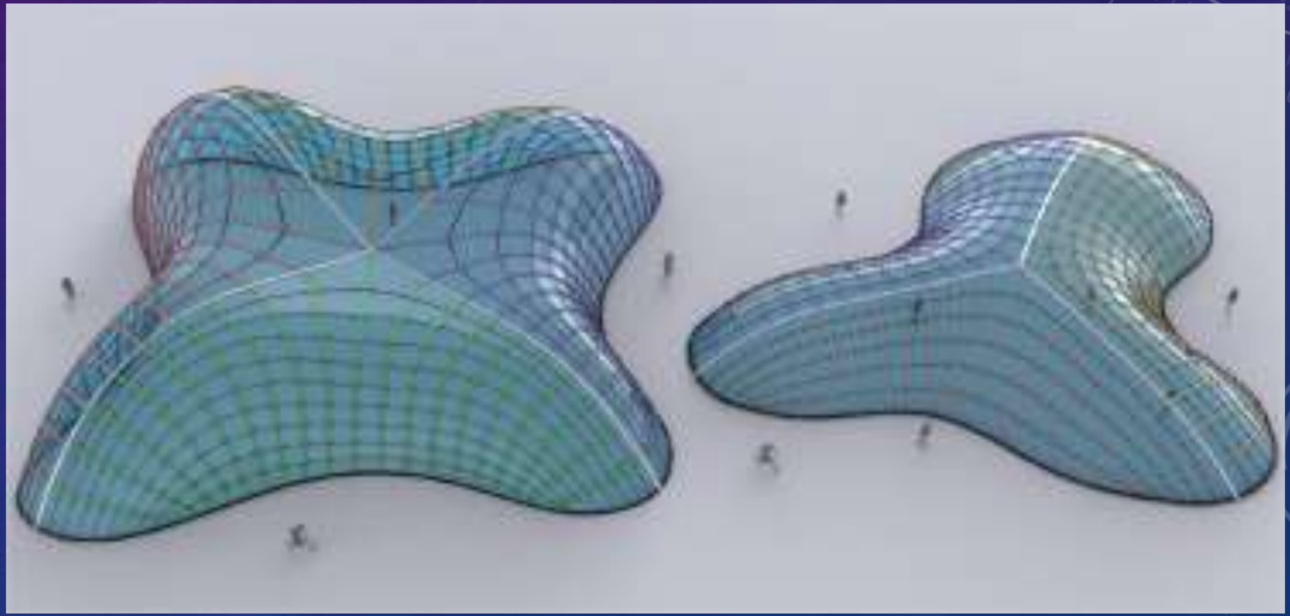
Applications

- Fitting B-spline surfaces
- Simulation
- Texture atlas
- Modeling & design



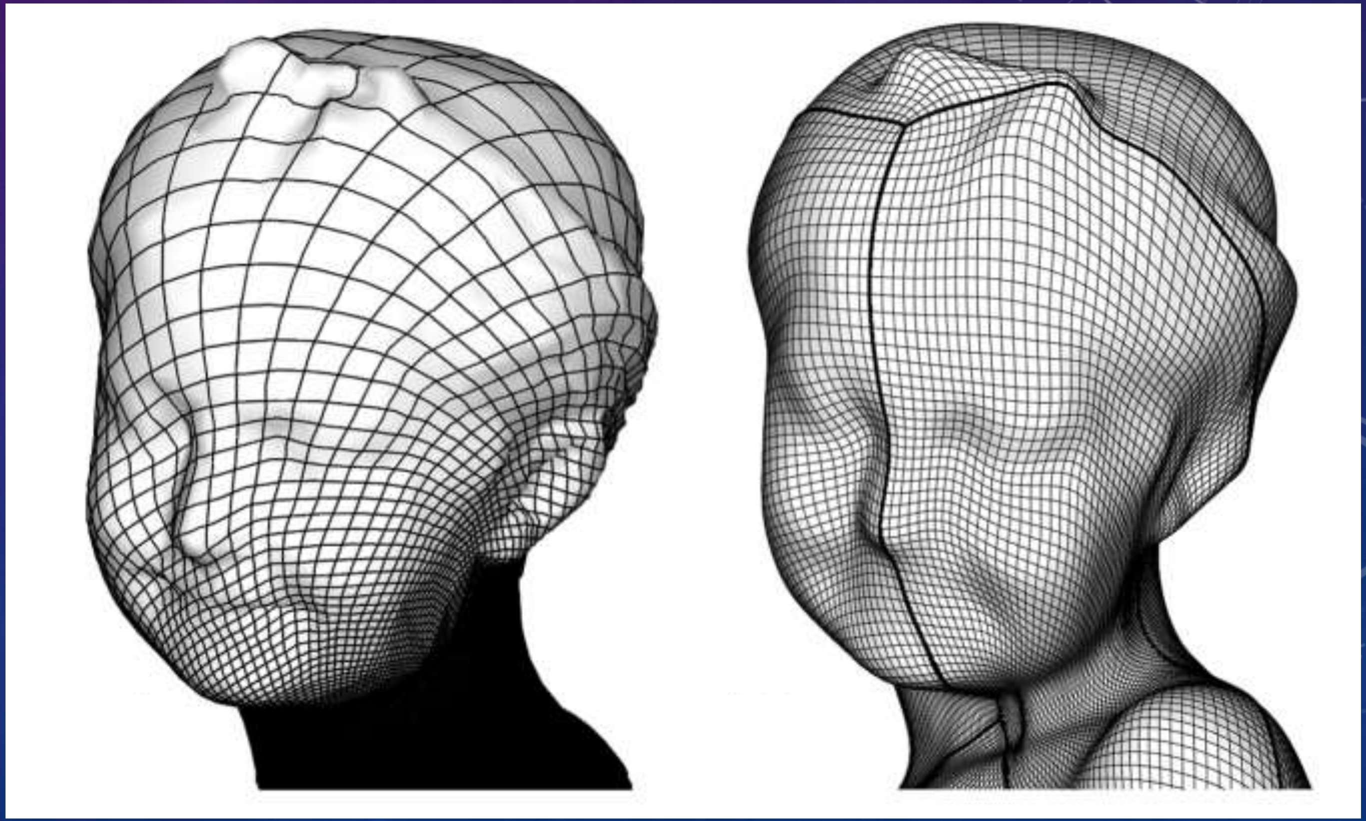
Applications

- Fitting B-spline surfaces
- Simulation
- Texture atlas
- Modeling & design
- Architecture



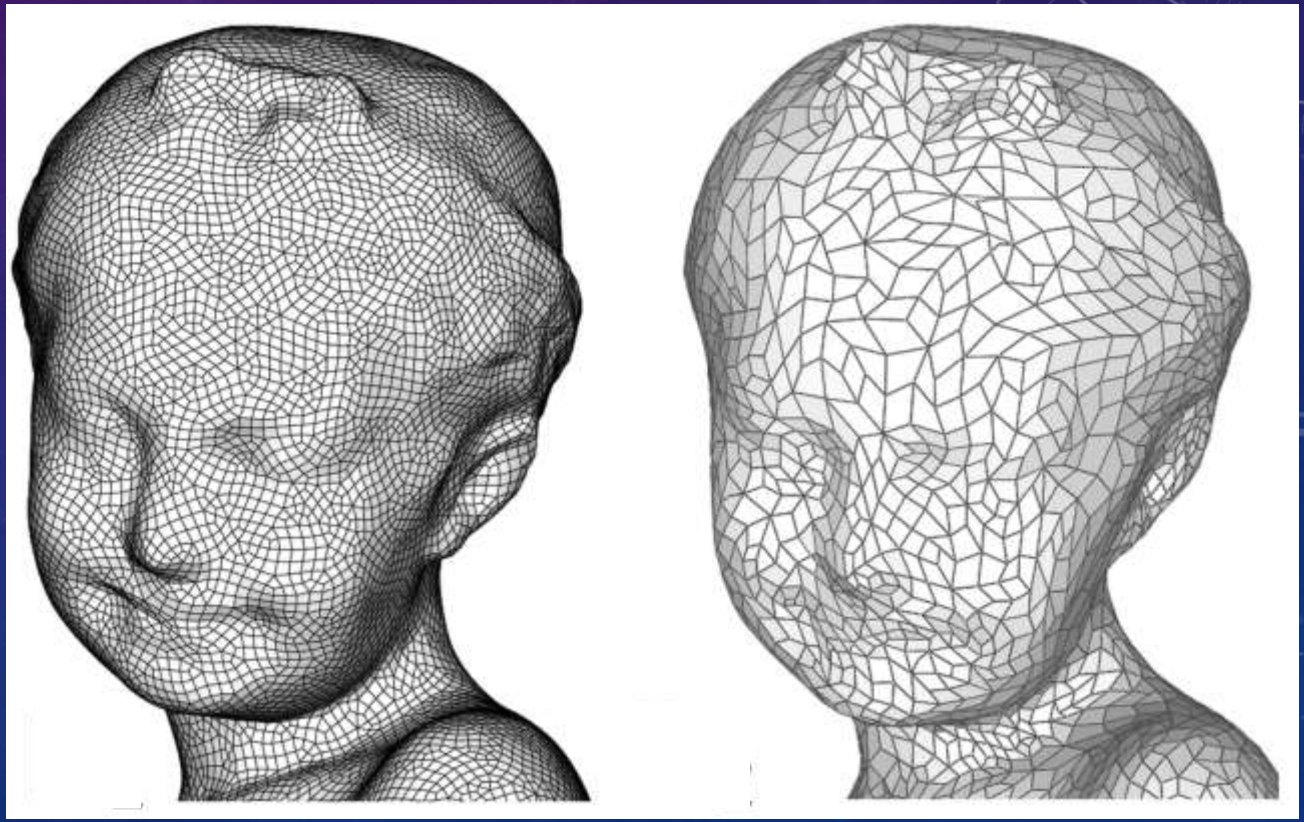
Requirements

- Lengths/angles distribution



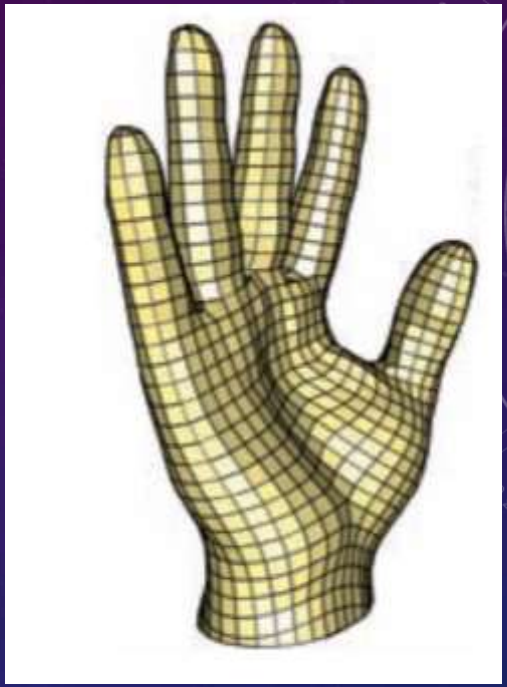
Requirements

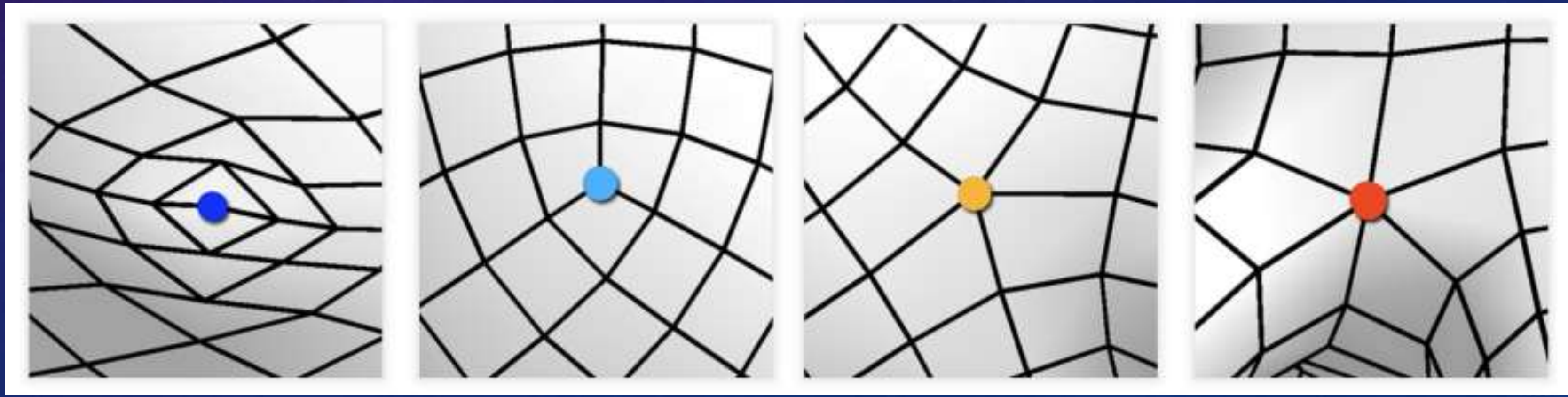
- Lengths/angles distribution
- Orthogonality



Requirements

- Lengths/angles distribution
- Orthogonality
- Regularity

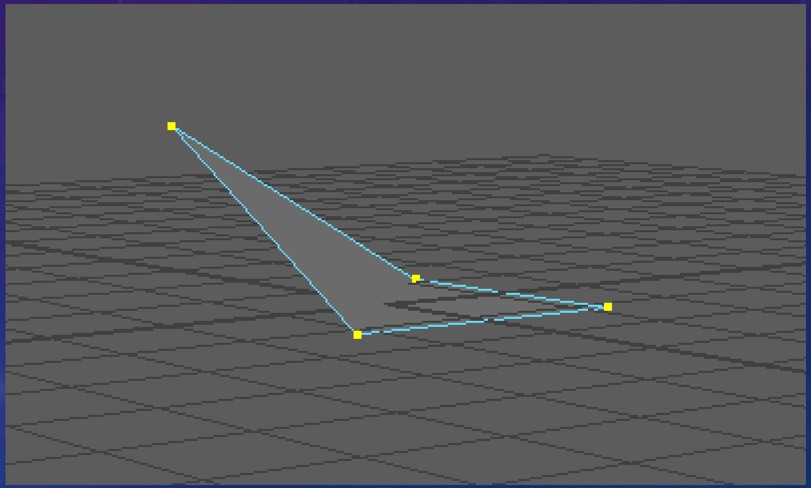




Valence: 2 3 5 6

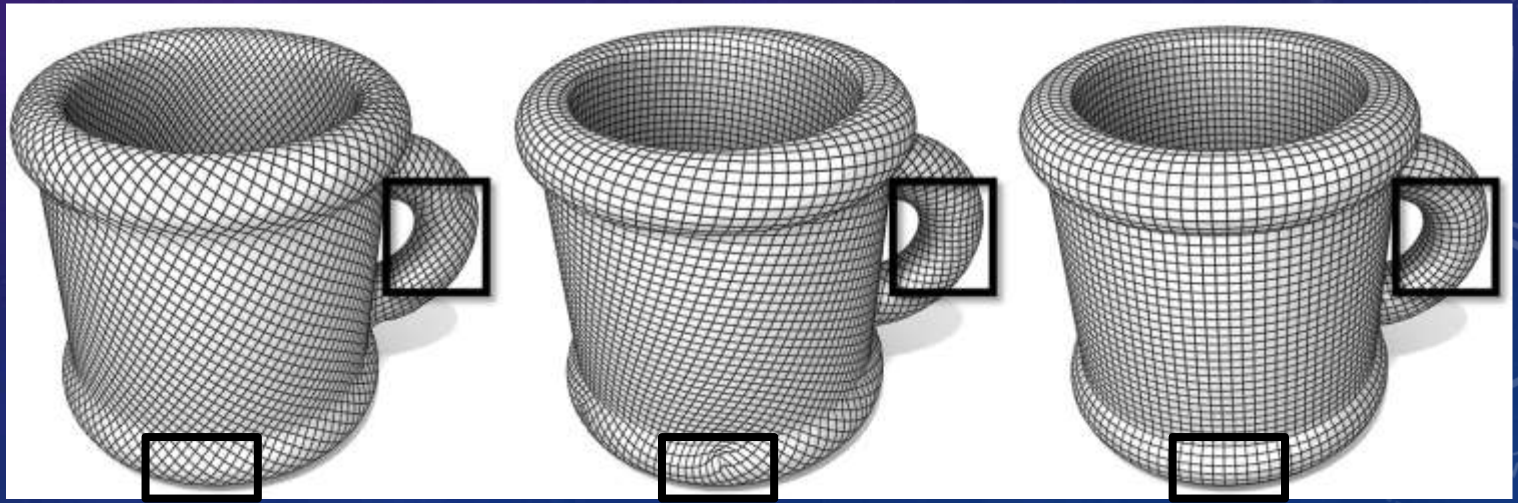
Requirements

- Lengths/angles distribution
- Orthogonality
- Regularity
- Planarity



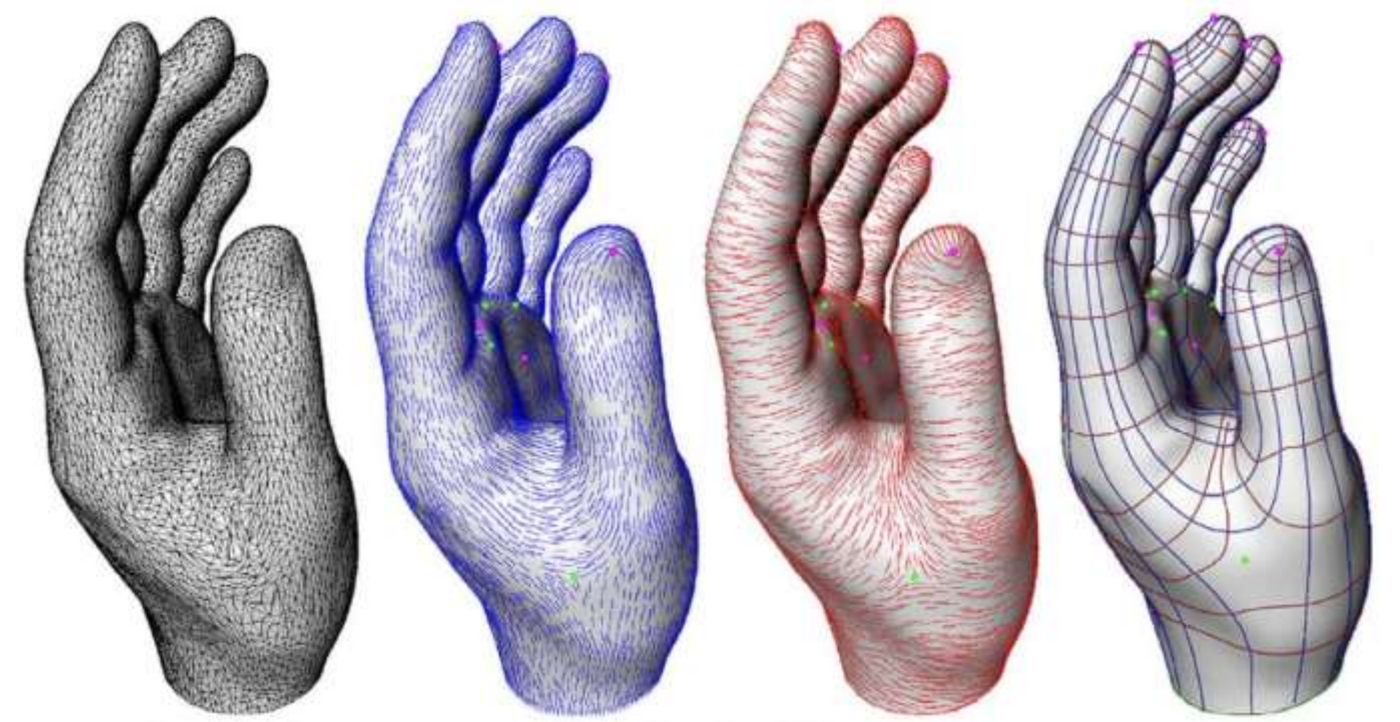
Requirements

- Lengths/angles distribution
- Orthogonality
- Regularity
- Planarity
- Feature alignment



Methods

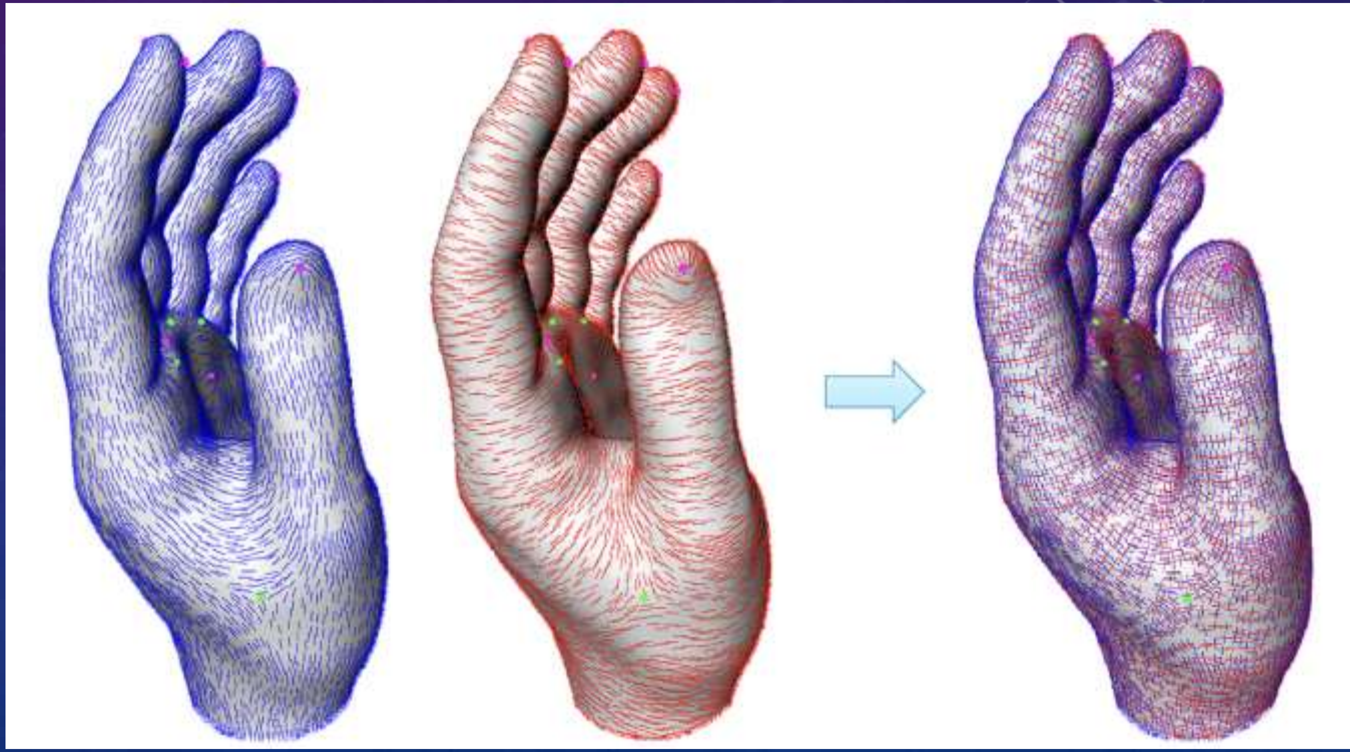
- Structure from curve tracing



Anisotropic Polygonal Remeshing [Alliez et al., Siggraph 2003]

Anisotropic Polygonal Remeshing

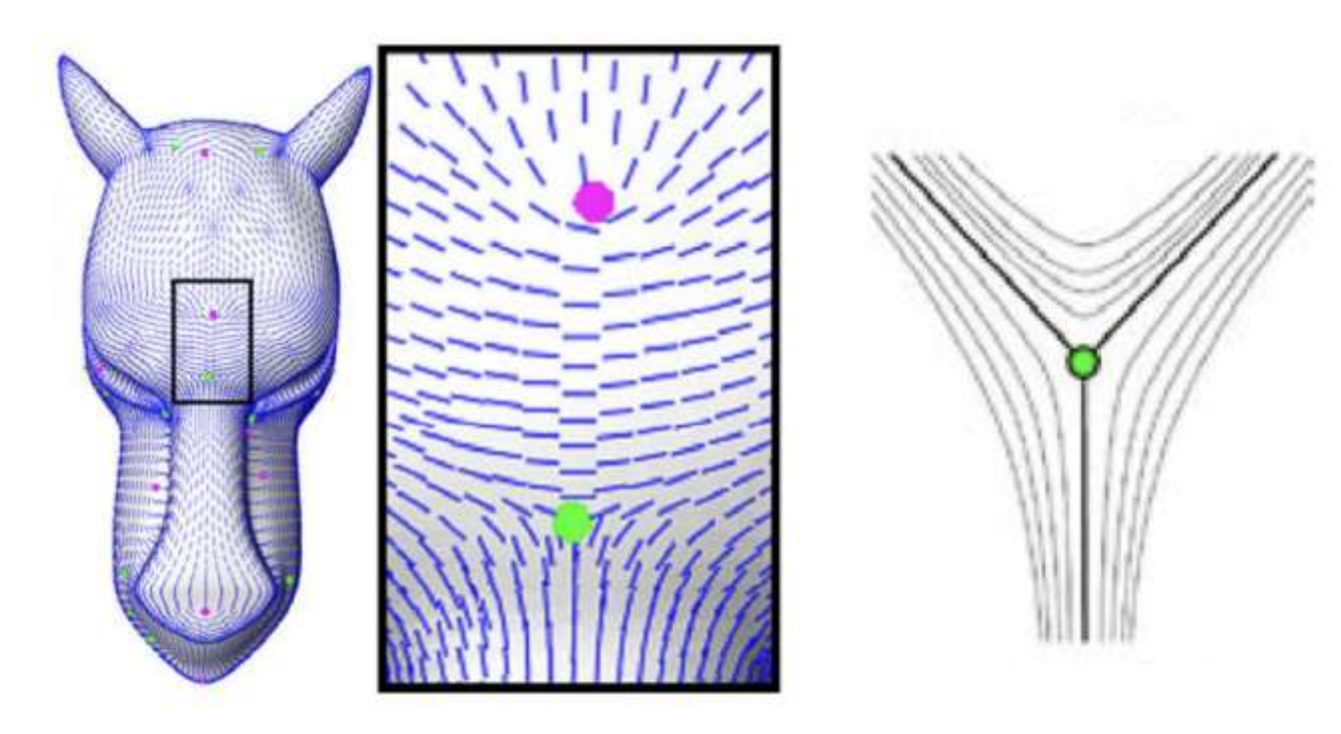
- Compute curvature directions



Min curvature Max curvature Cross Field

Anisotropic Polygonal Remeshing

- Compute curvature directions



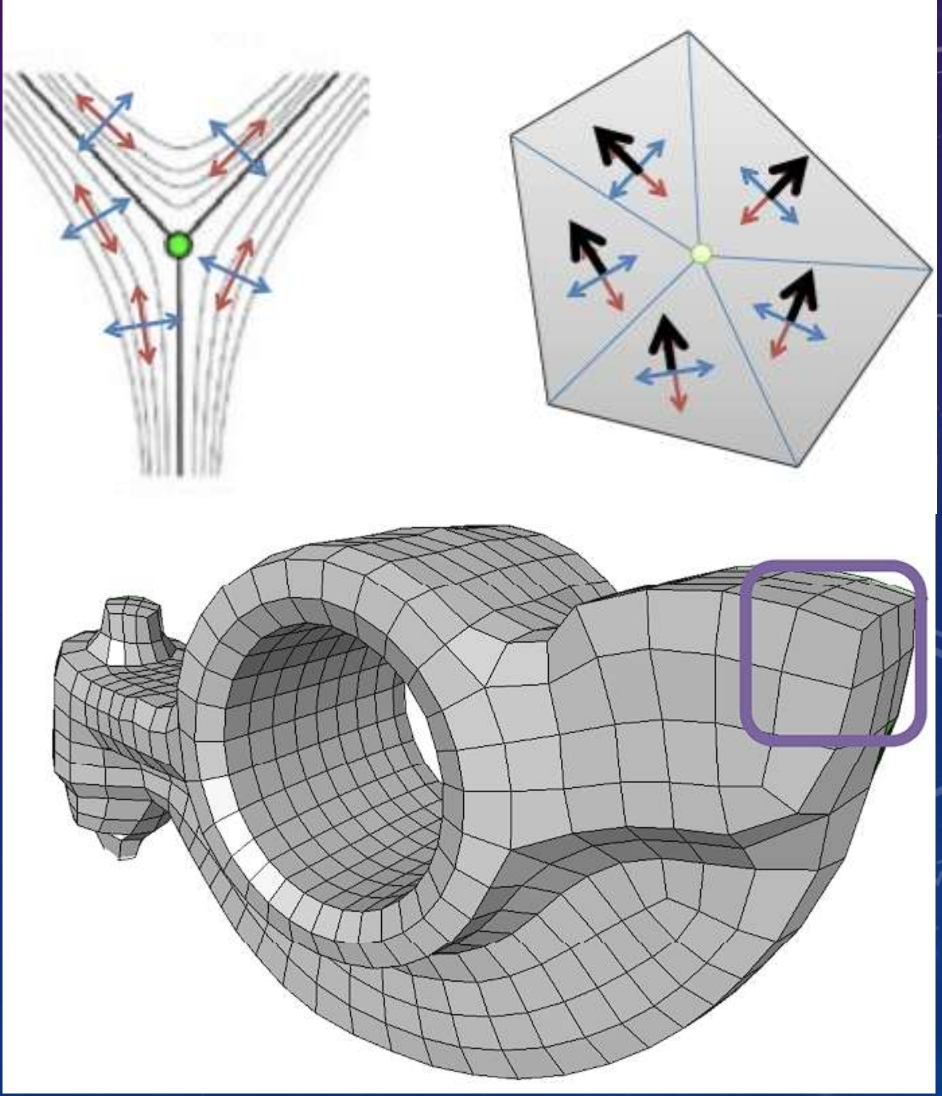
Umbilics: minimal curvature = maximal curvature

Anisotropic Polygonal Remeshing

- Compute curvature directions

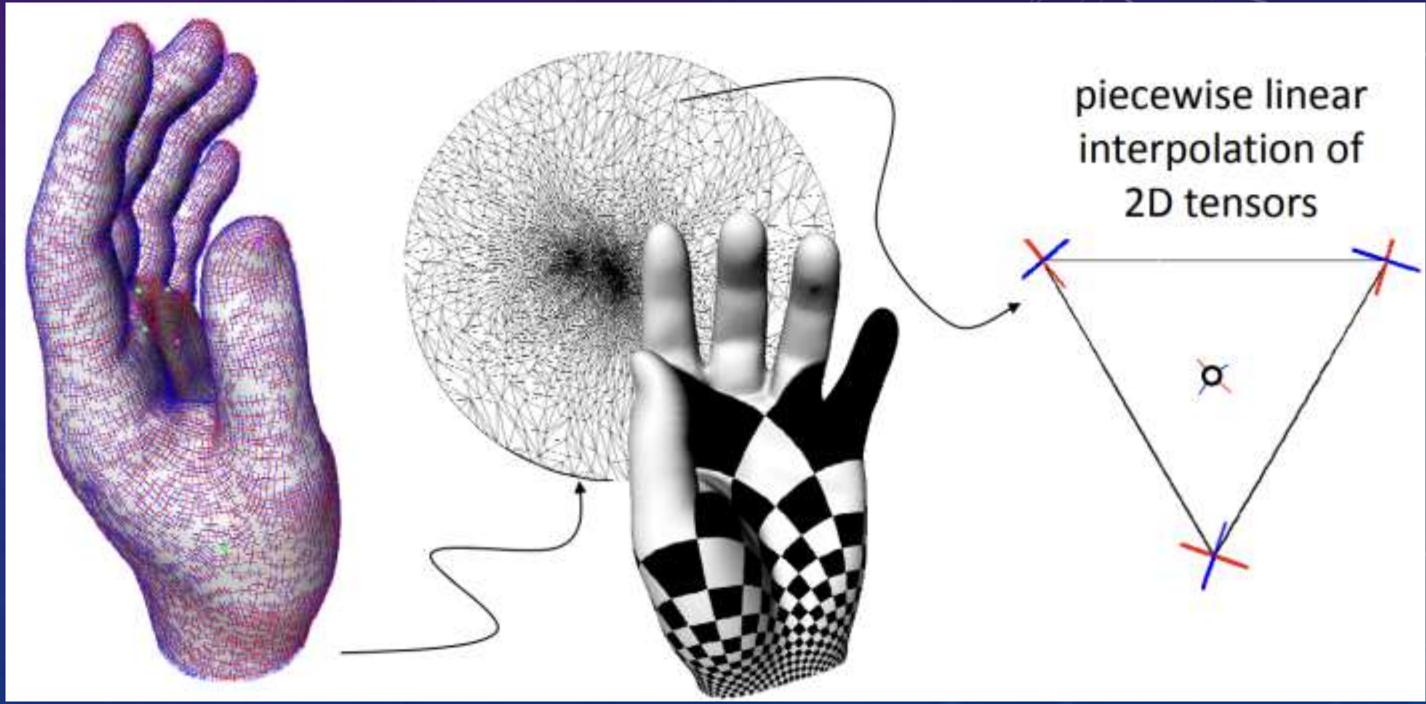
Umbilics generate singularities in cross field

There is no consistent selection of 1 direction which gives a smooth vector field



Anisotropic Polygonal Remeshing

- Compute curvature directions
- Find Umbilics/Singularities

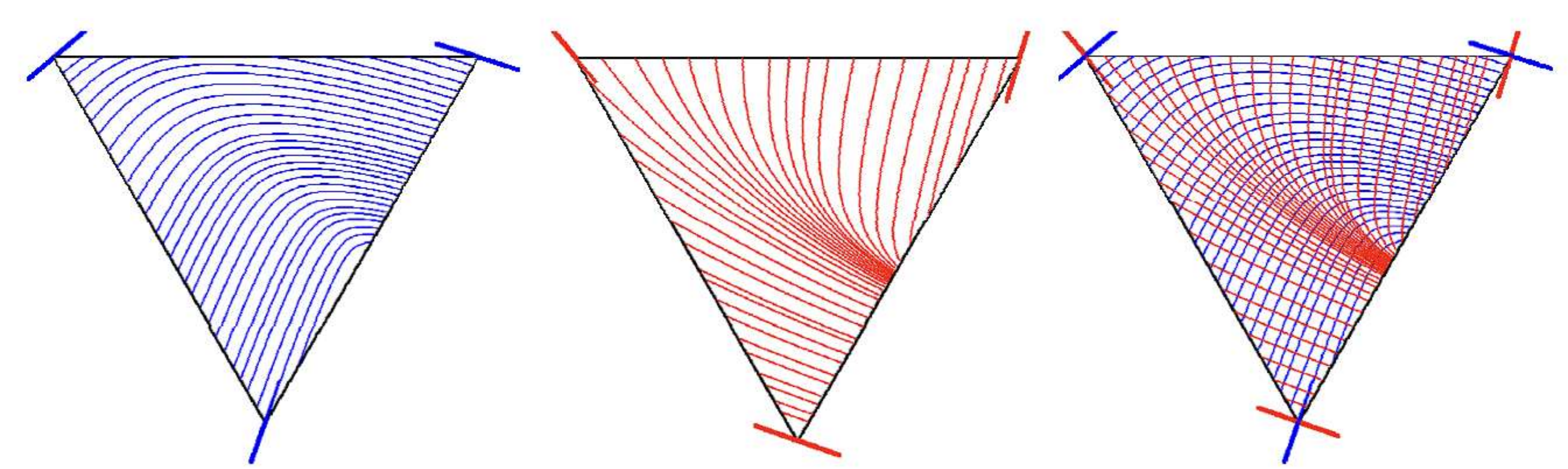


Conformal parameterization

2D tensor field using barycentric coordinates

Anisotropic Polygonal Remeshing

- Compute curvature directions
- Find Umbilics/Singularities



Regular case Minor directions Major directions Both directions

Anisotropic Polygonal Remeshing

- Compute curvature directions
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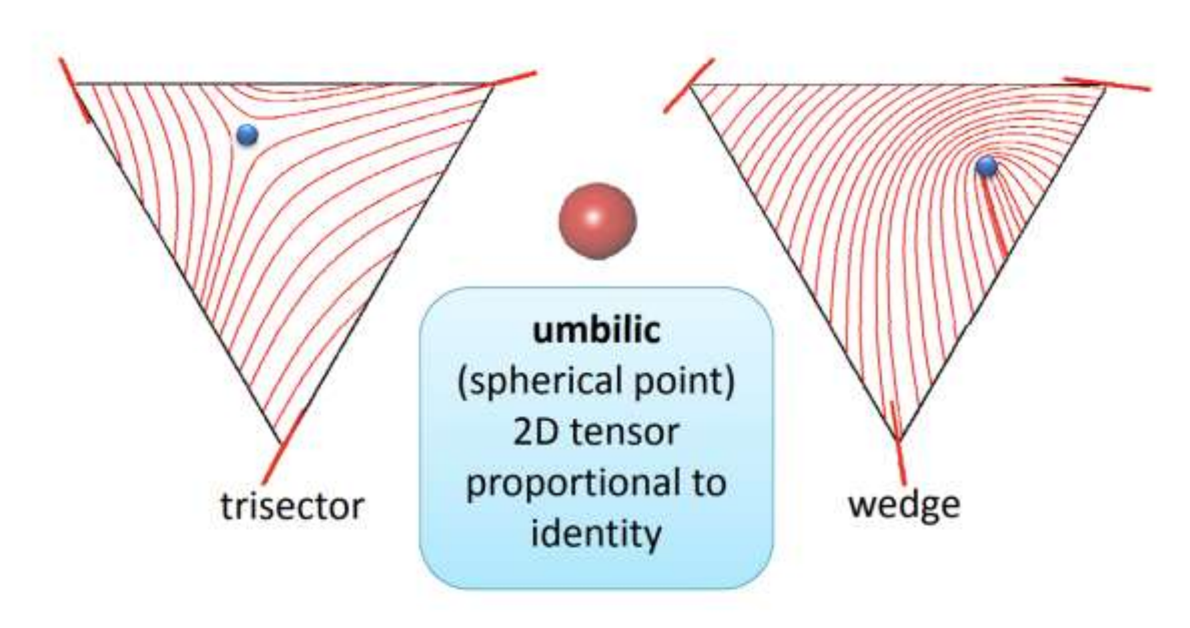
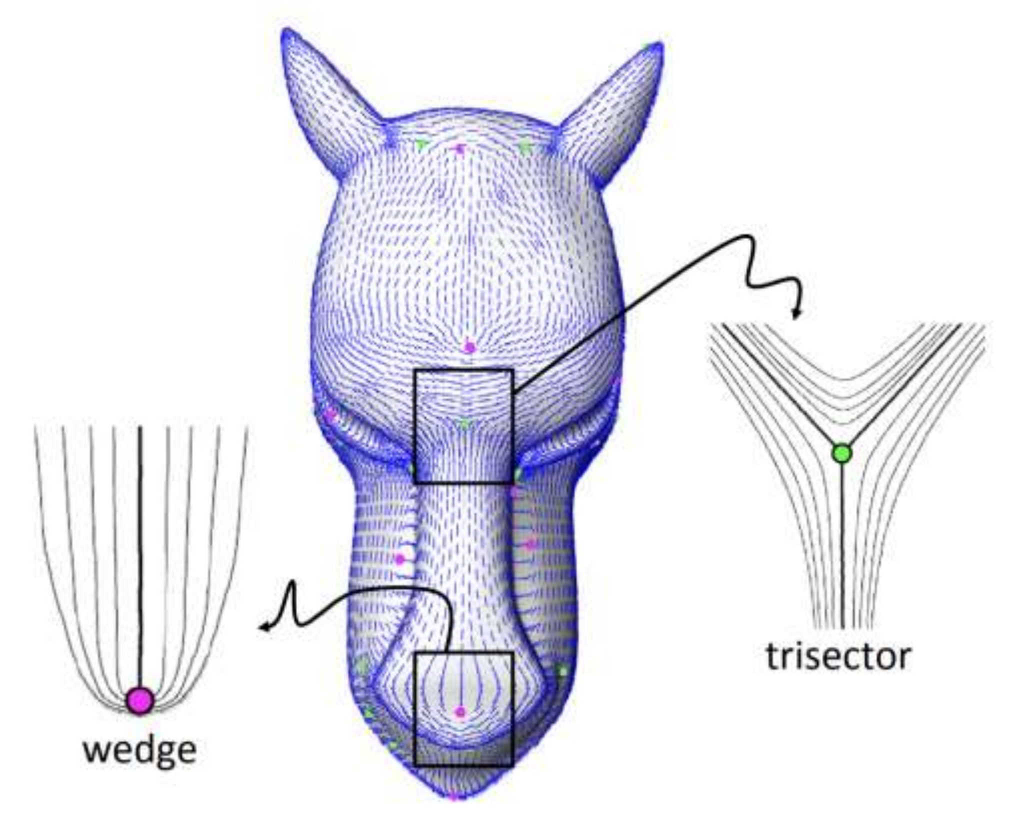
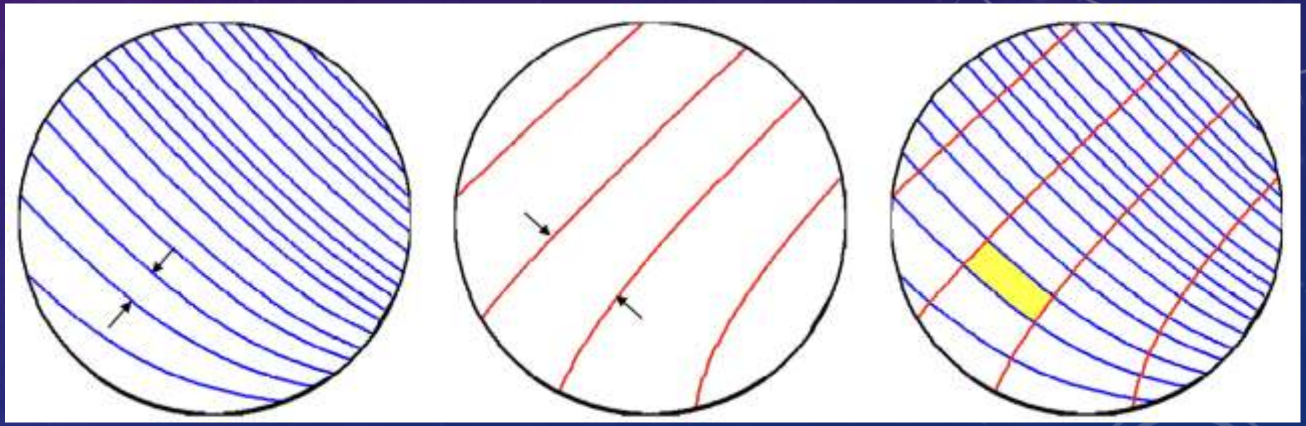




Anisotropic Polygonal Remeshing

- Compute curvature directions
- Find Umbilics/Singularities
- Trace curvature lines
  1. Generate curves tangent to cross field
  2. Intersection of curves → vertices of quad mesh

Minor net   Major net   Overlay



Anisotropic Polygonal Remeshing

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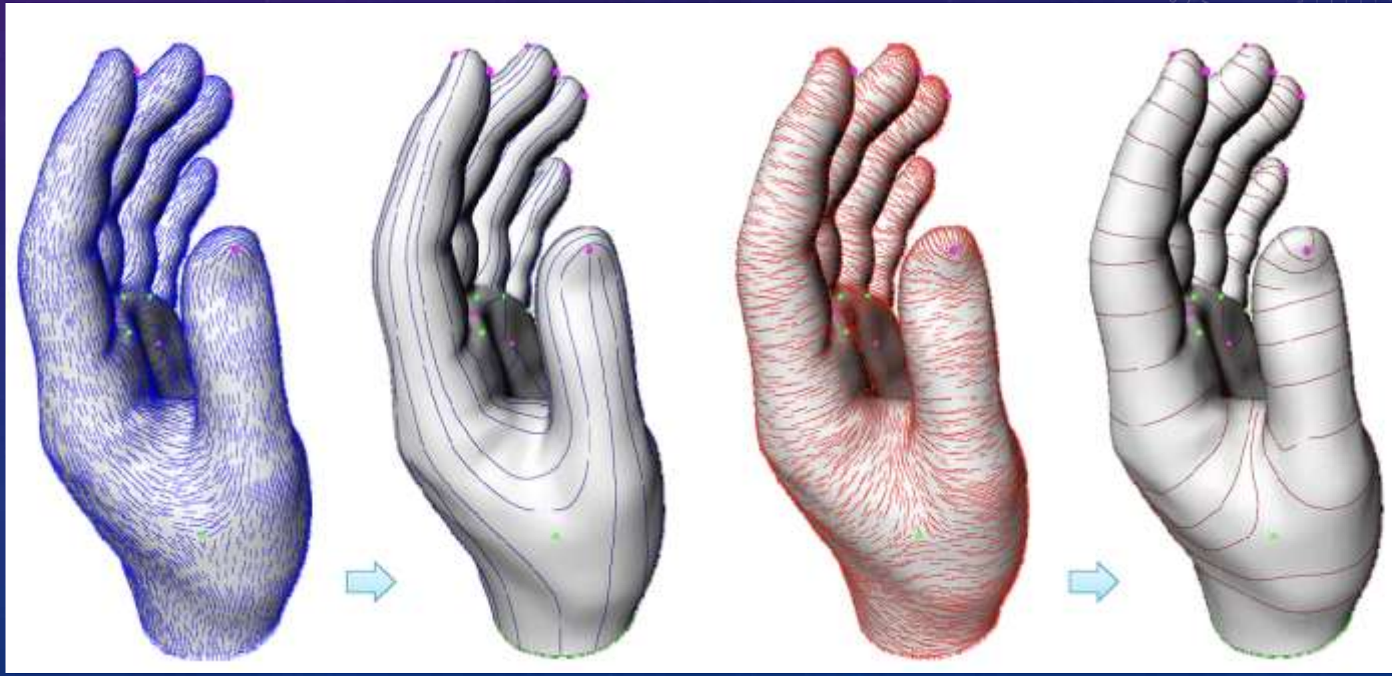
Minor net

Major net



Anisotropic Polygonal Remeshing

- Compute curvature directions
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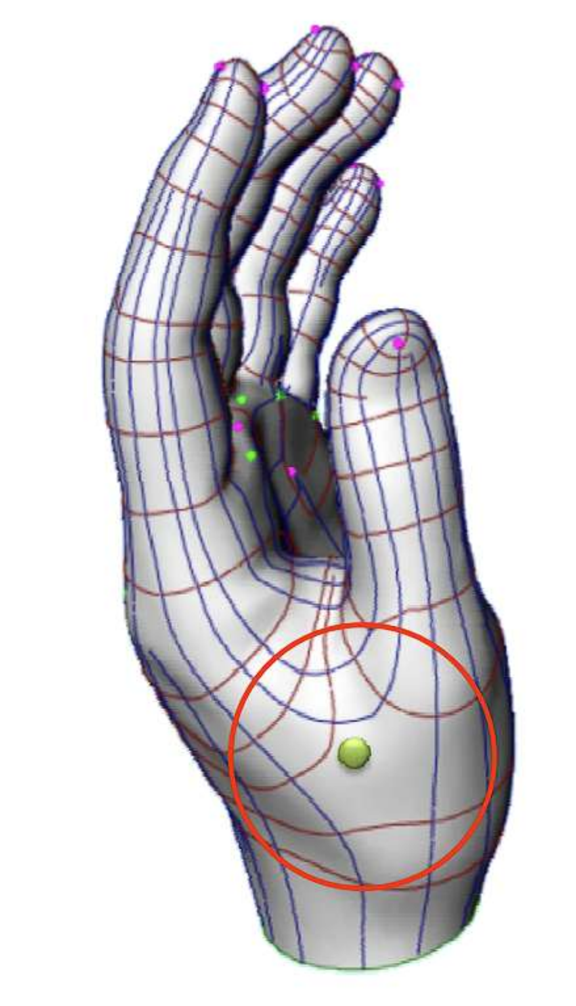
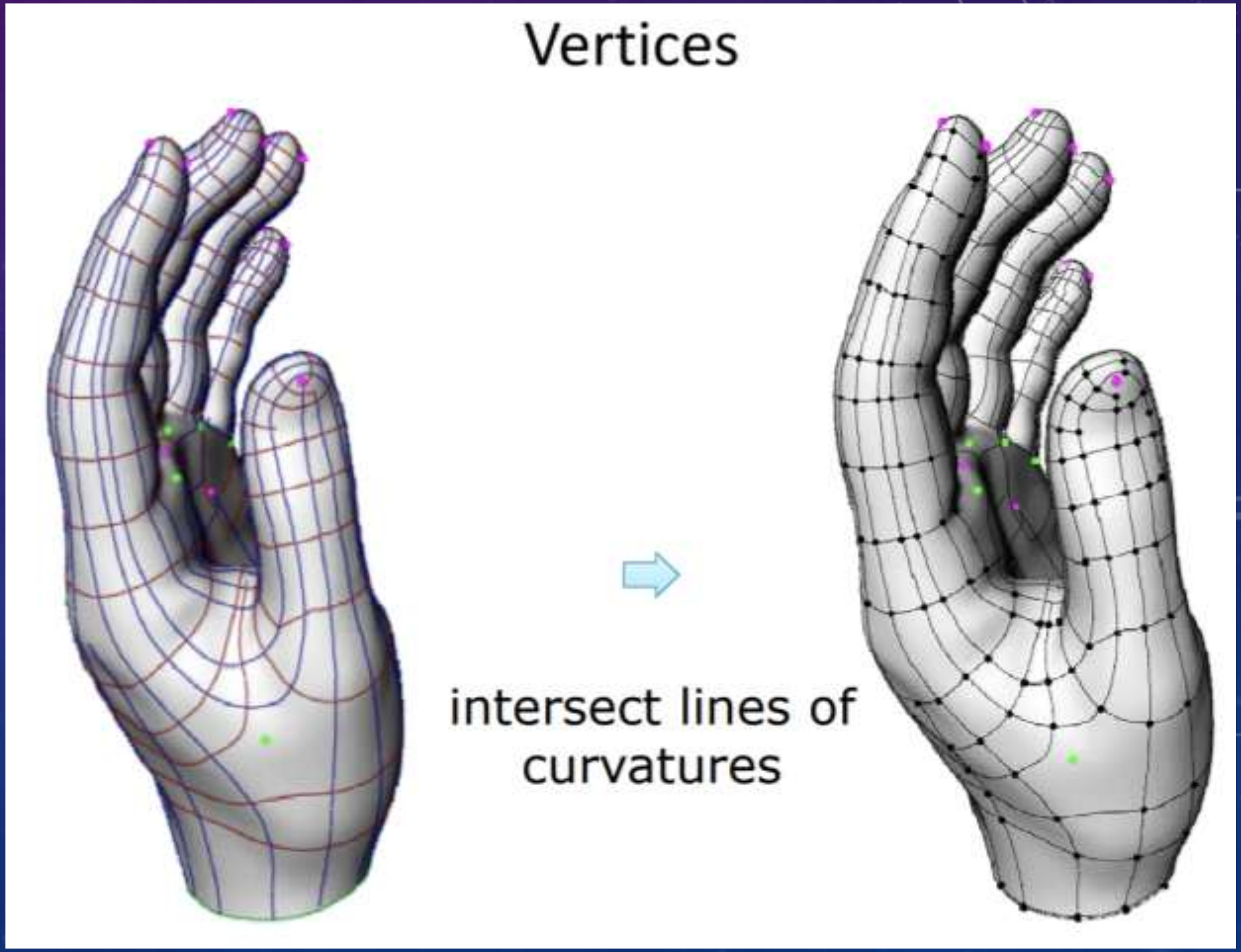


Add umbilic points in isotropic regions

Anisotropic Polygonal Remeshing

- Compute curvature directions
- Find Umbilics/Singularities
- Trace curvature lines
- Overlay
- Meshing



Anisotropic Polygonal Remeshing

- Compute curvature directions
- Find Umbilics/Singularities
- Trace curvature lines
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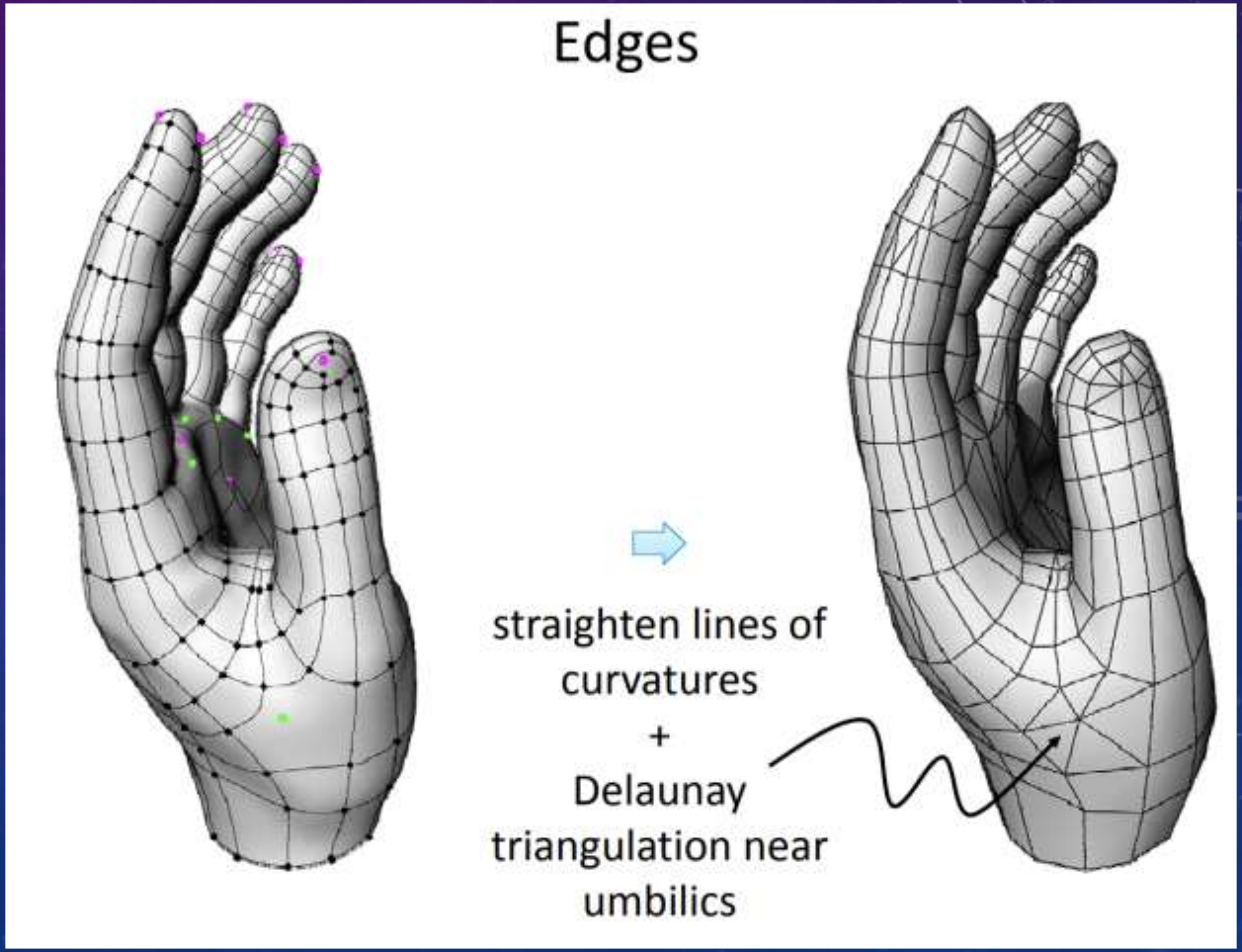
- Meshing



Anisotropic Polygonal Remeshing

- Compute curvature directions
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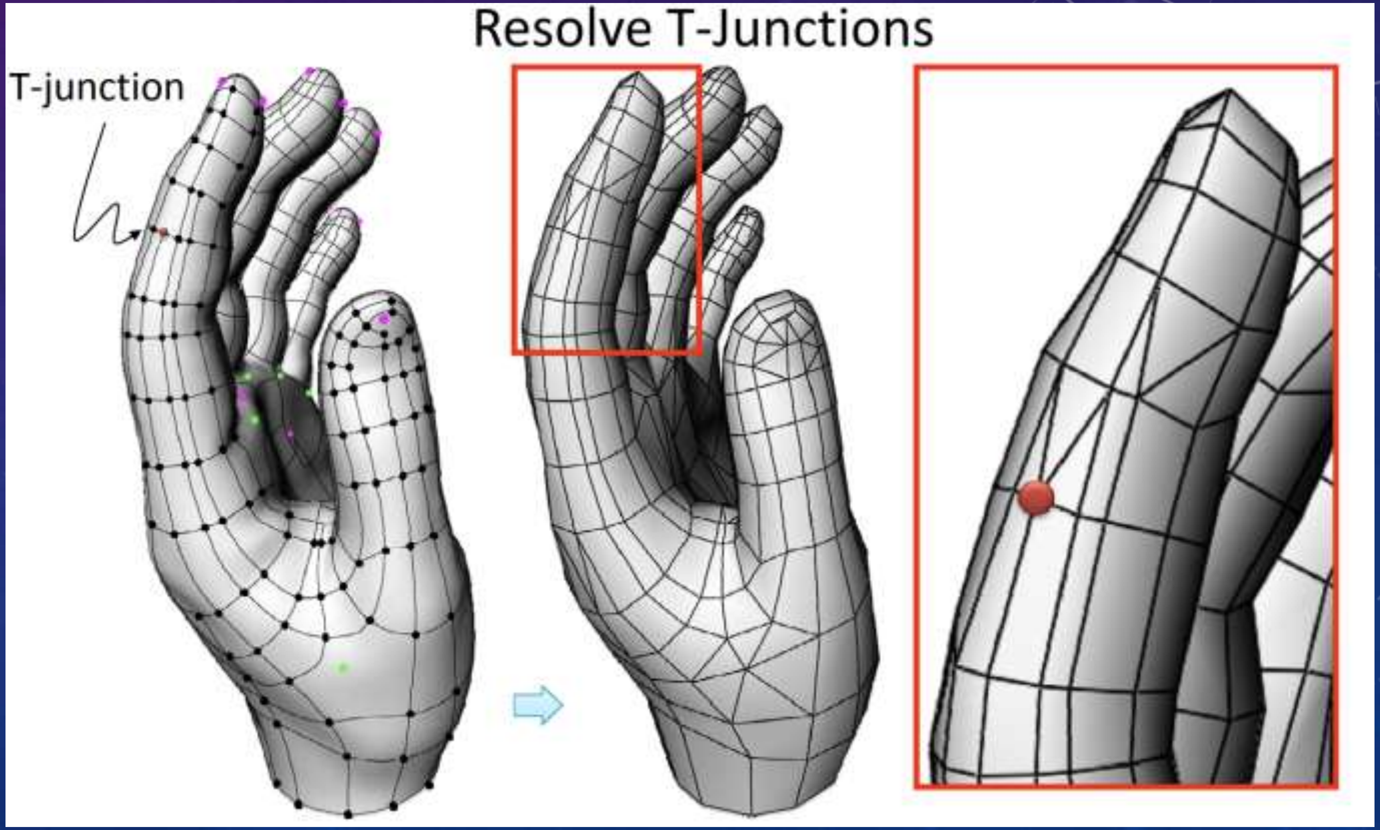
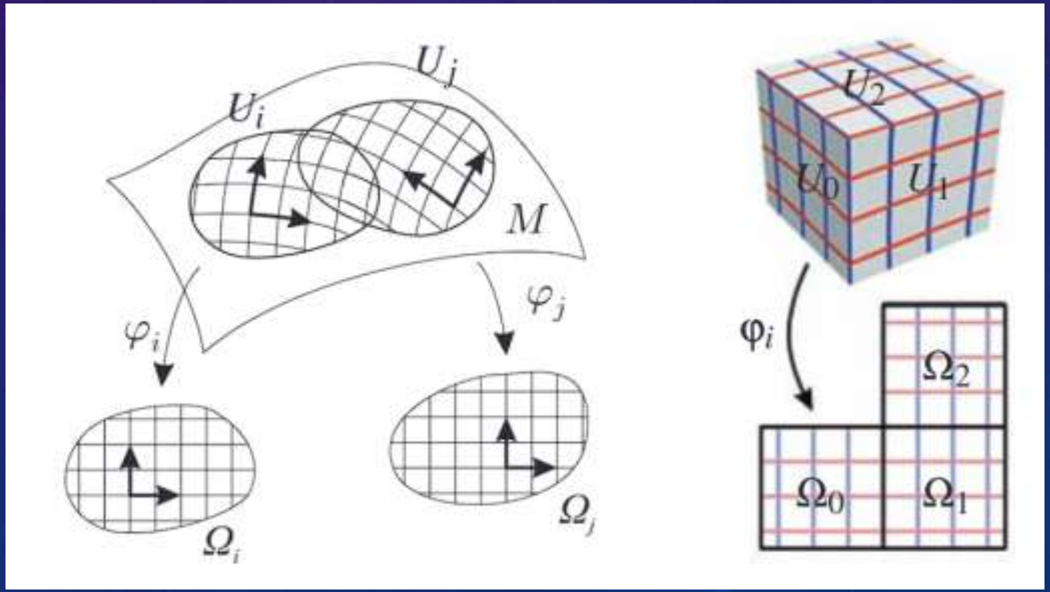


Methods

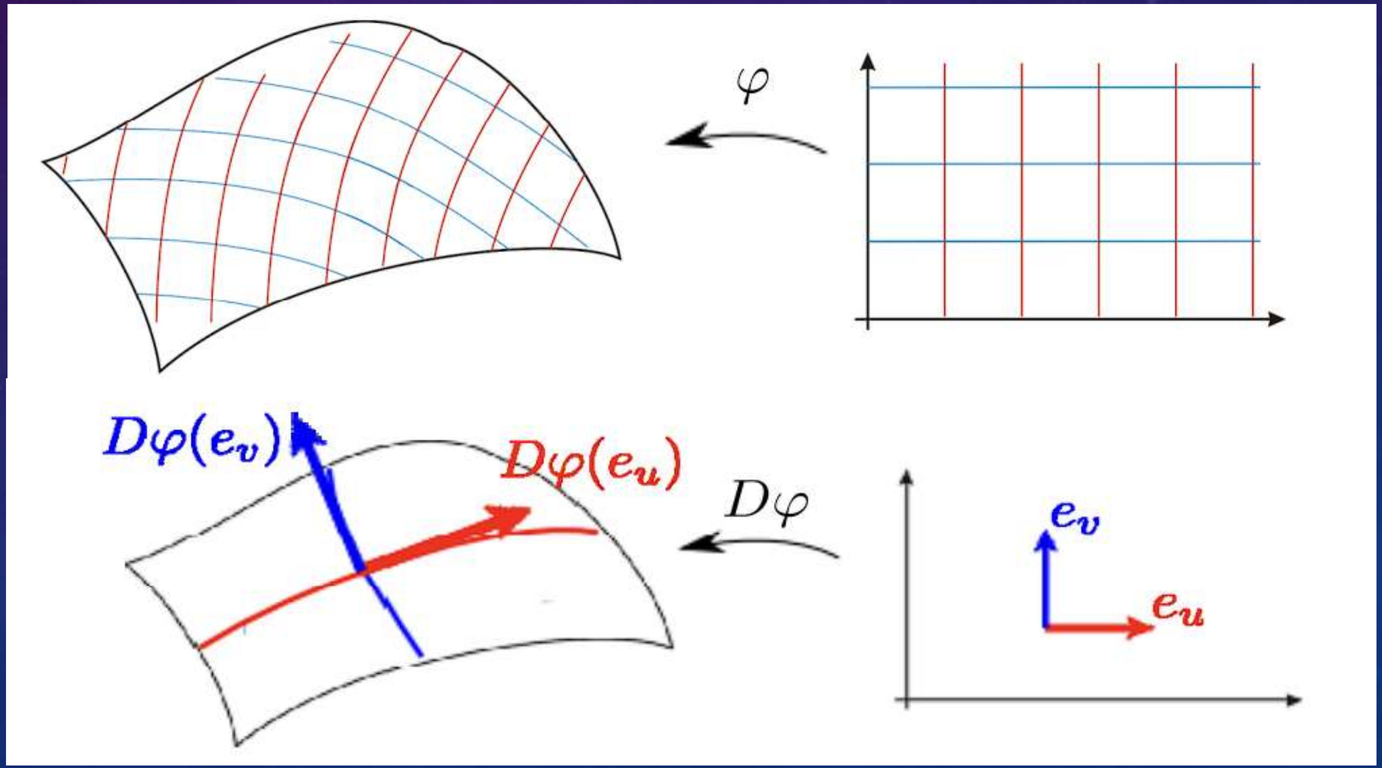
- Structure from curve tracing
- Parameterization from cross fields



QuadCover [Kälberer et al., EuroGraphics 2007]

Parameterization from cross fields

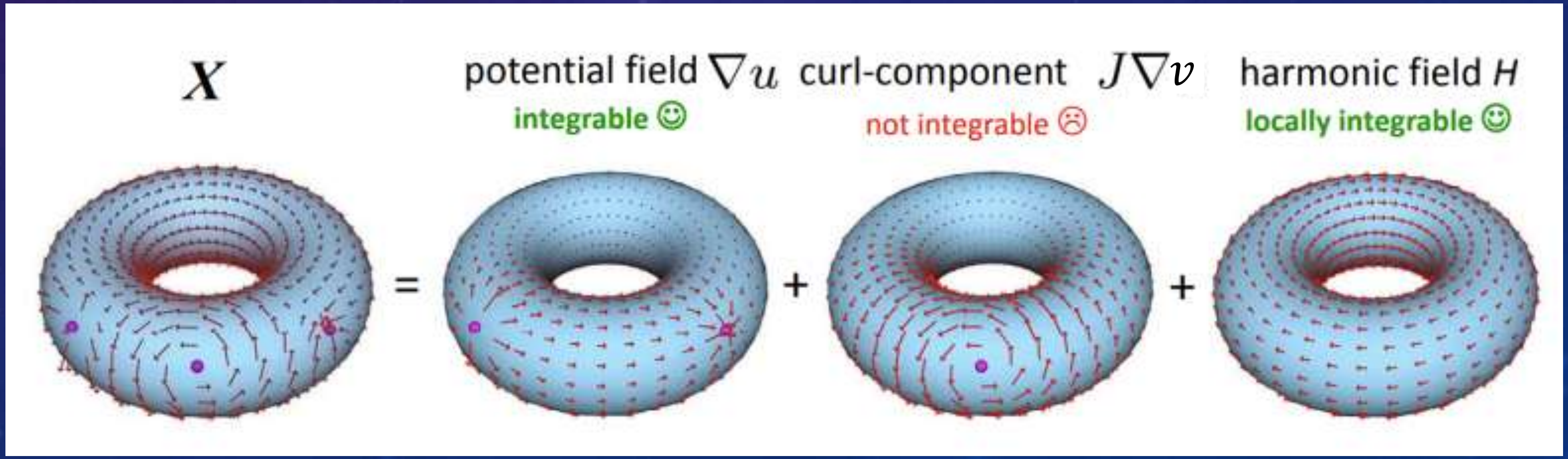
- Function $\xrightarrow{\text{differential}}$ Gradient vector fields $\xrightarrow{\text{integration}}$ Function



Parameterization from cross fields

- Integrability: given f find g, s.t. $f = \nabla g$
- Only possible if f is "integrable"

A vector field U is locally integrable iff $\nabla \times U = 0$

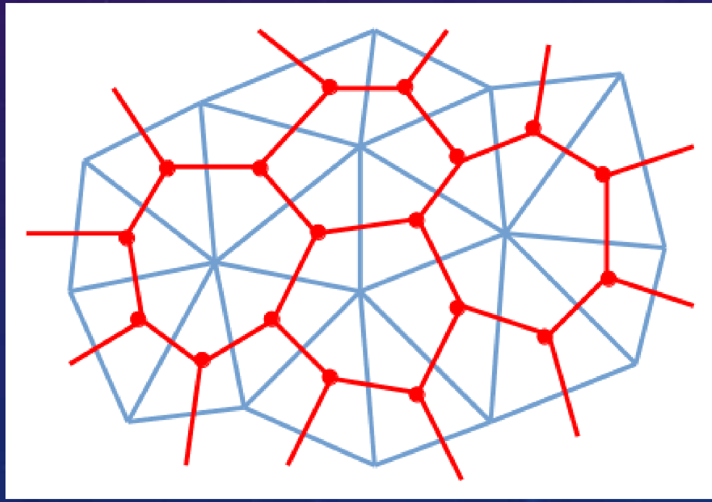


Hodge-Helmholtz Decomposition

Parameterization from cross fields

- Potential field : $\min_u \int \|\nabla u - X\|^2 \rightarrow \Delta u = -\nabla \cdot X$
- Curl-component: $\min_u \int \|\mathcal{J}\nabla v - X\|^2 \rightarrow [\Delta v]_{ij} = -[\nabla \times X]_{ij}$

Edge-based Poisson equation



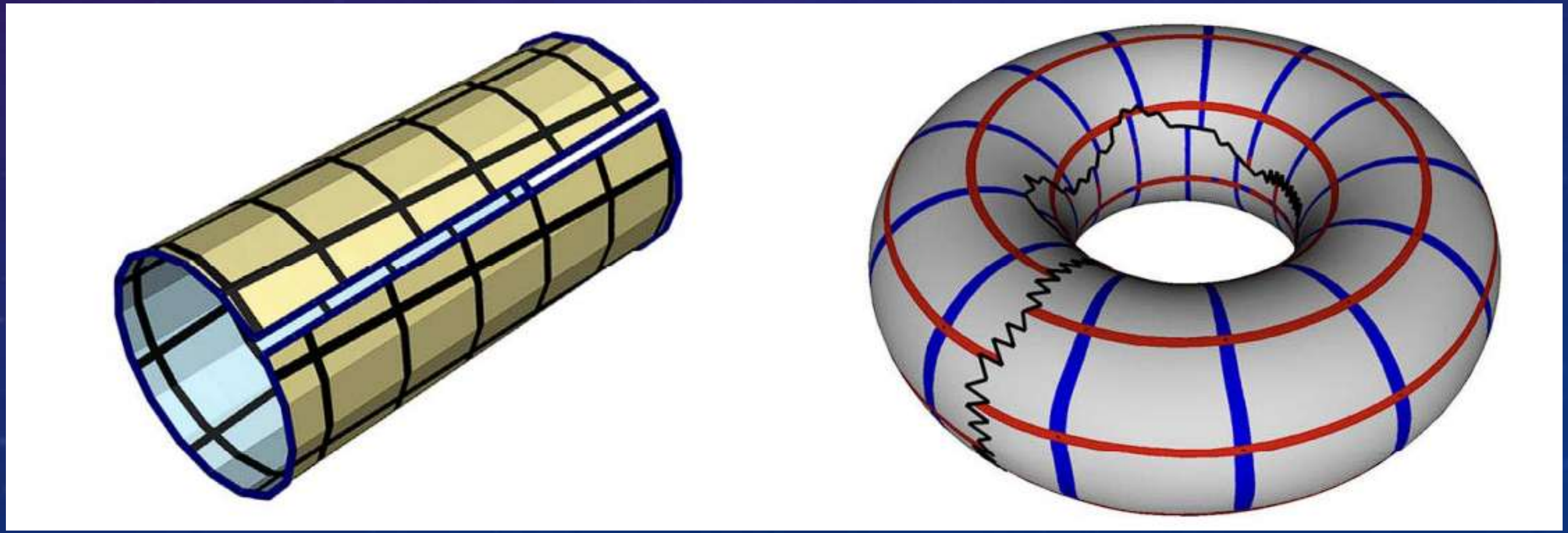
Dual mesh

QuadCover

- Assure local integrability of input cross fields $X = (X_1, X_2)$

$$Y_1 = X_1 - \mathcal{J}\nabla v_1, \qquad Y_2 = X_2 - \mathcal{J}\nabla v_2$$

- Assure global continuity of Y along Homology gens



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- Assure local integrability of input cross fields $X = (X_1, X_2)$

$$Y_1 = X_1 - \mathcal{J}\nabla v_1, \qquad Y_2 = X_2 - \mathcal{J}\nabla v_2$$

- Assure global continuity of $Y + H$ along Homology gens

1. Compute Homology generators $\gamma_1, \ldots, \gamma_{2g}$ (= basis of all closed loops)
2. Measure mismatch $\int_{\gamma_i} Y_1 ds \in \mathbb{R}$, $\int_{\gamma_i} Y_2 ds \in \mathbb{R}$
3. Compute L_2 smallest harmonic vector fields $H_j, j = 1,2$ s.t. $\int_{\gamma_i} (Y_j + H_j) ds \in \mathbb{Z}$

QuadCover

- Assure local integrability of input cross fields $X = (X_1, X_2)$

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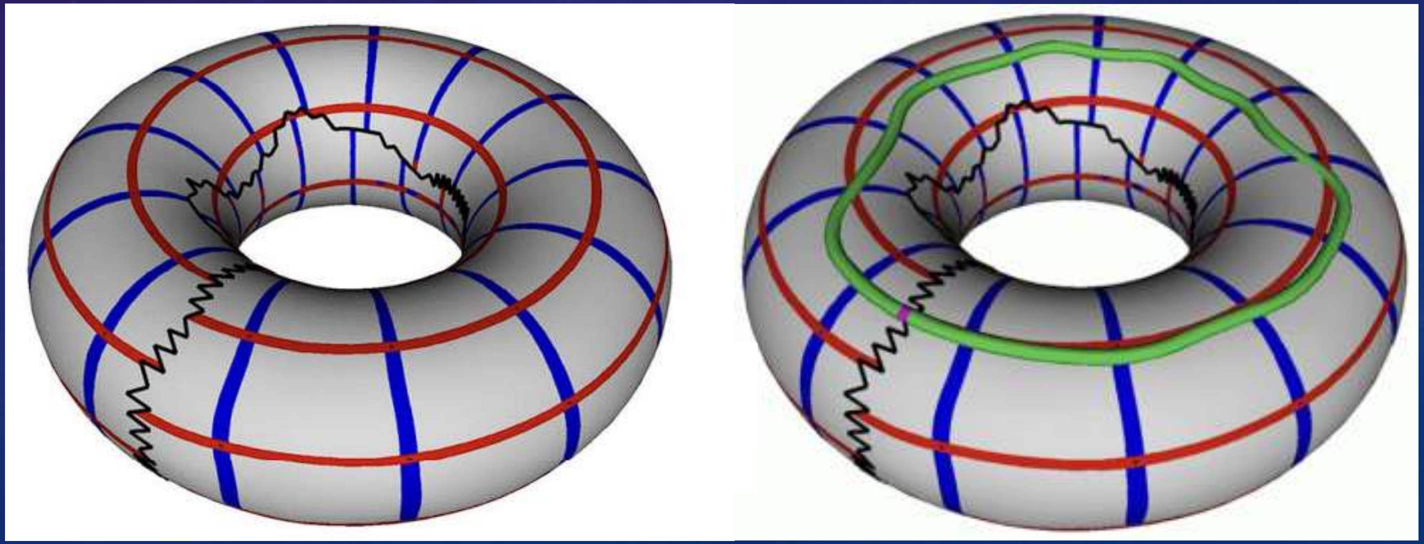


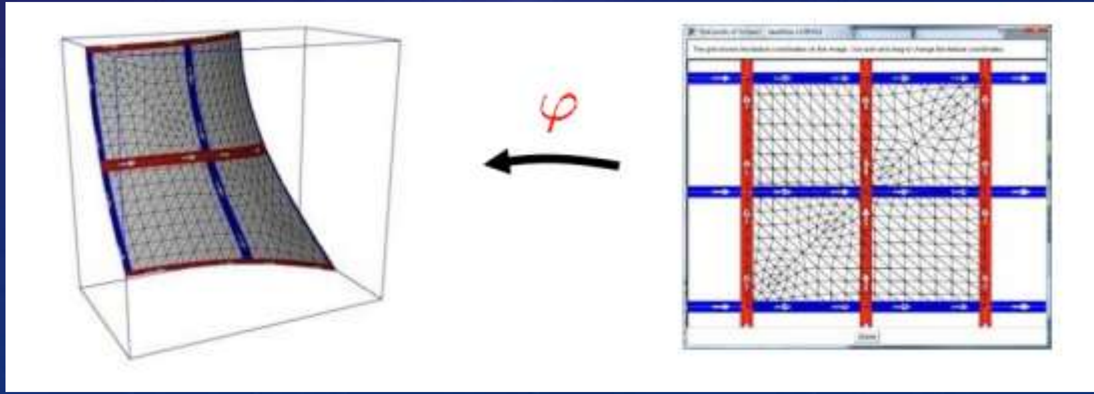
QuadCover

- Assure local integrability of input cross fields $X = (X_1, X_2)$

$$Y_1 = X_1 - \mathcal{J}\nabla v_1, \qquad Y_2 = X_2 - \mathcal{J}\nabla v_2$$

- Assure global continuity of $Y + H$ along Homology gens

- Global integration of $Y + H$ on the mesh gives parameterization



Methods

- Structure from curve tracing
- Parameterization from cross fields
- Mix-integer optimization

1. Mixed-Integer Quadrangulation
2. Instant Field-Aligned Meshes
3. …