Mesh Parameterization III

USTC, 2024 Spring

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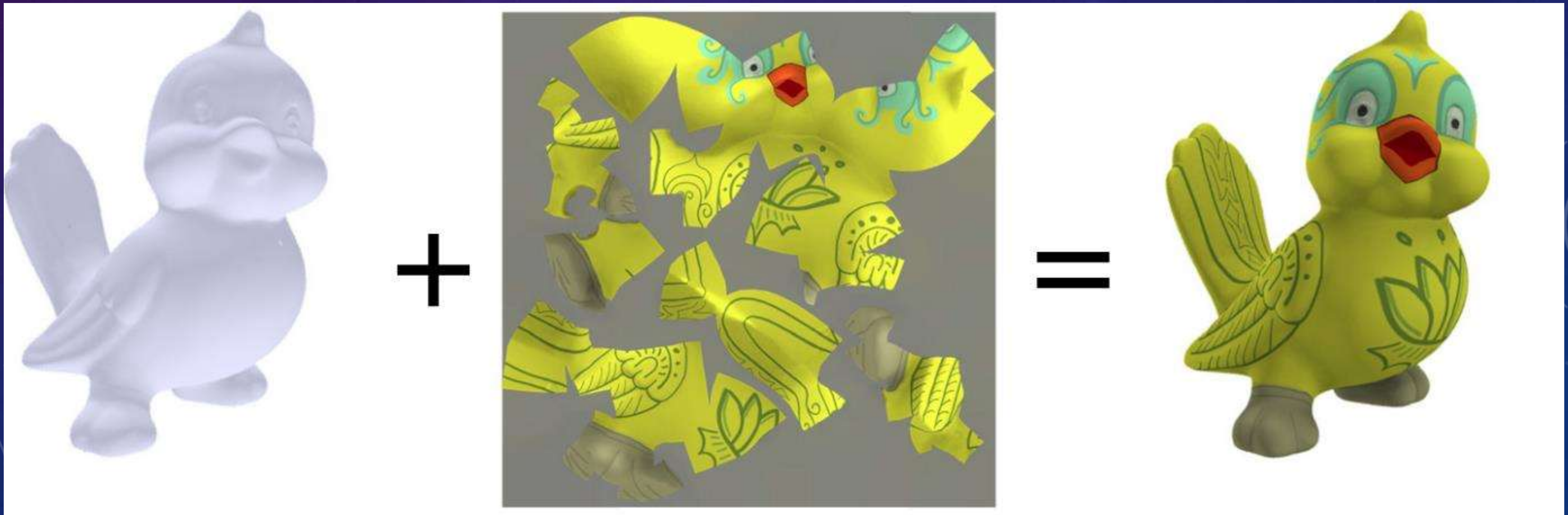
https://qingfang1208.github.io/

Applications

- Atlas generation
- Peeling art
- Meshing/remeshing
- Inter-surface mappings

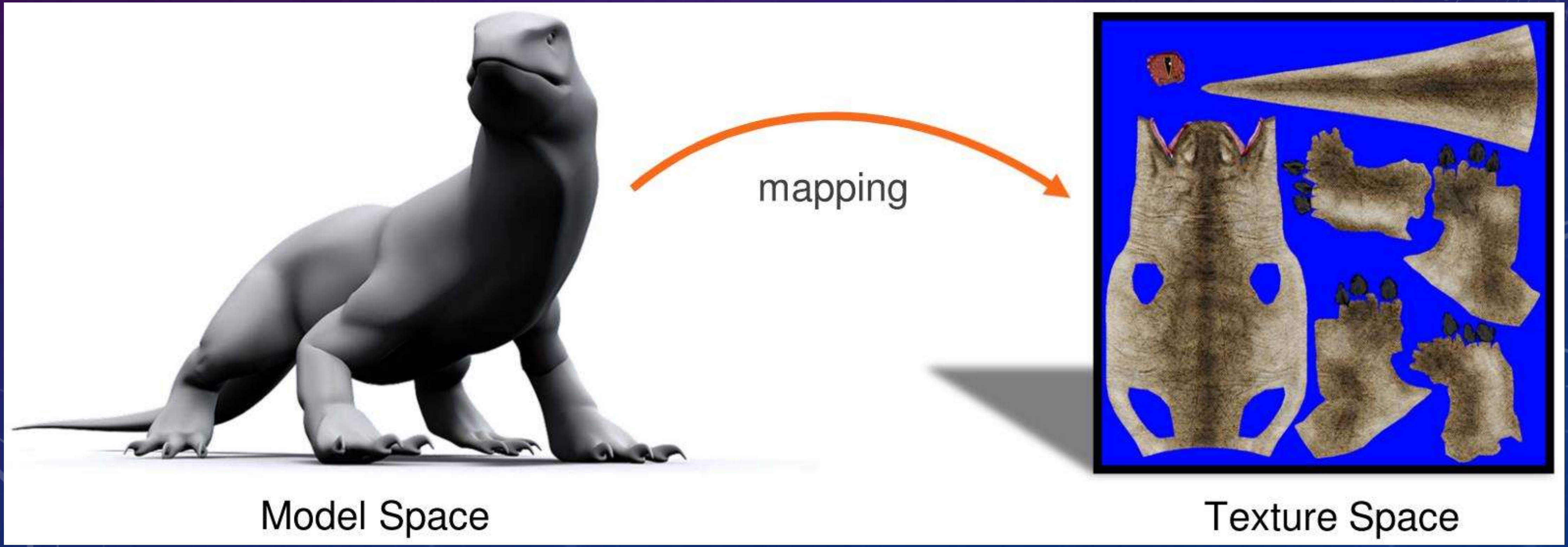
Texture mapping

- Texture mapping is a method for defining high frequency detail, surface texture, or color information on a computer-generated graphic or 3D model

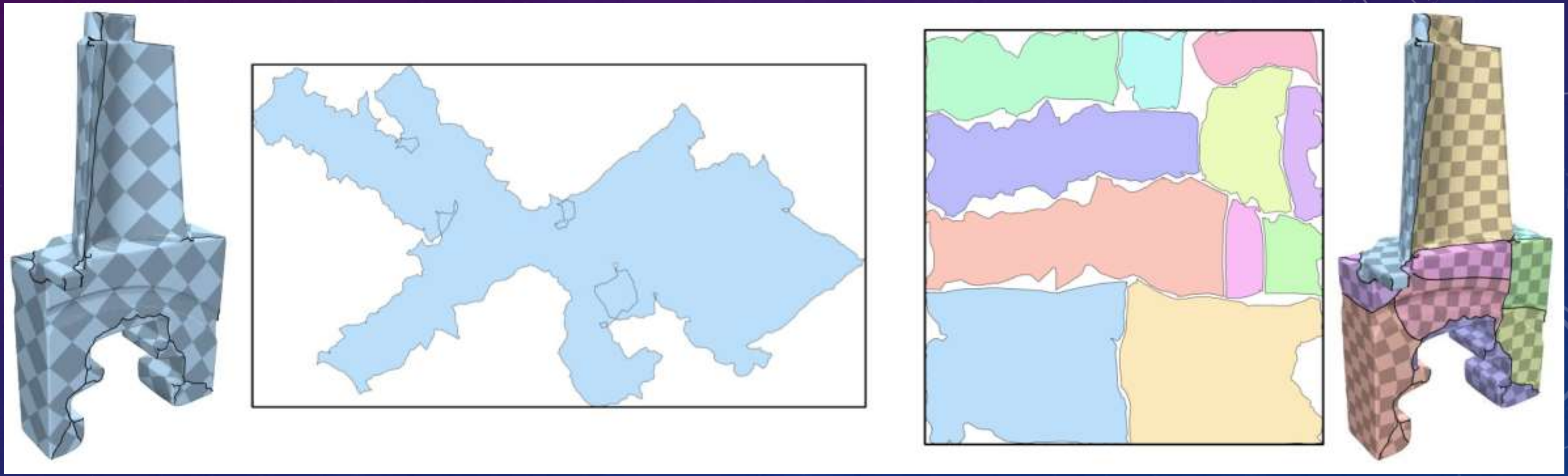


Atlas

- Requires defining a mapping from the model space to the texture space.



Generation process



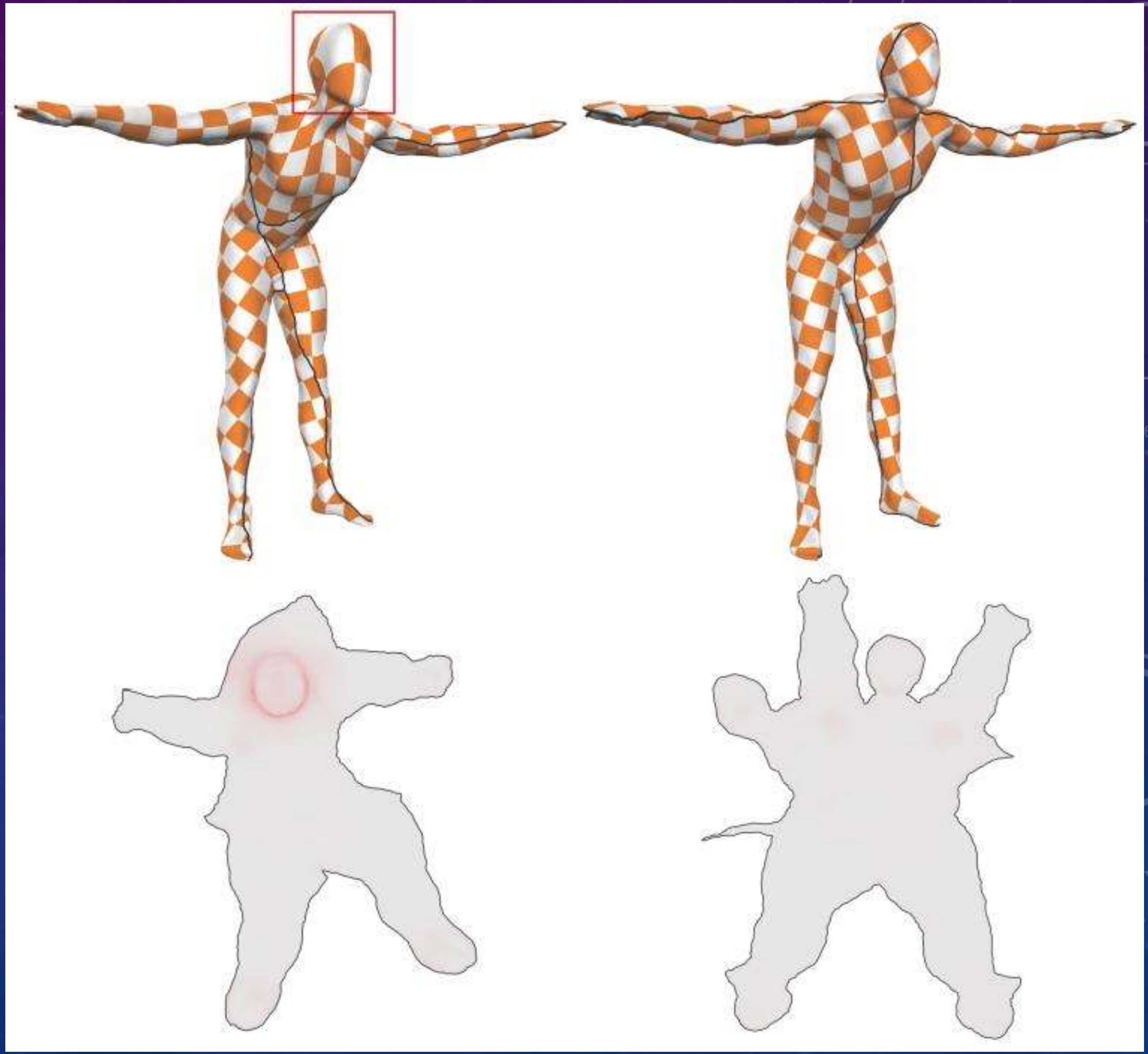
Mesh cutting

Parameterization

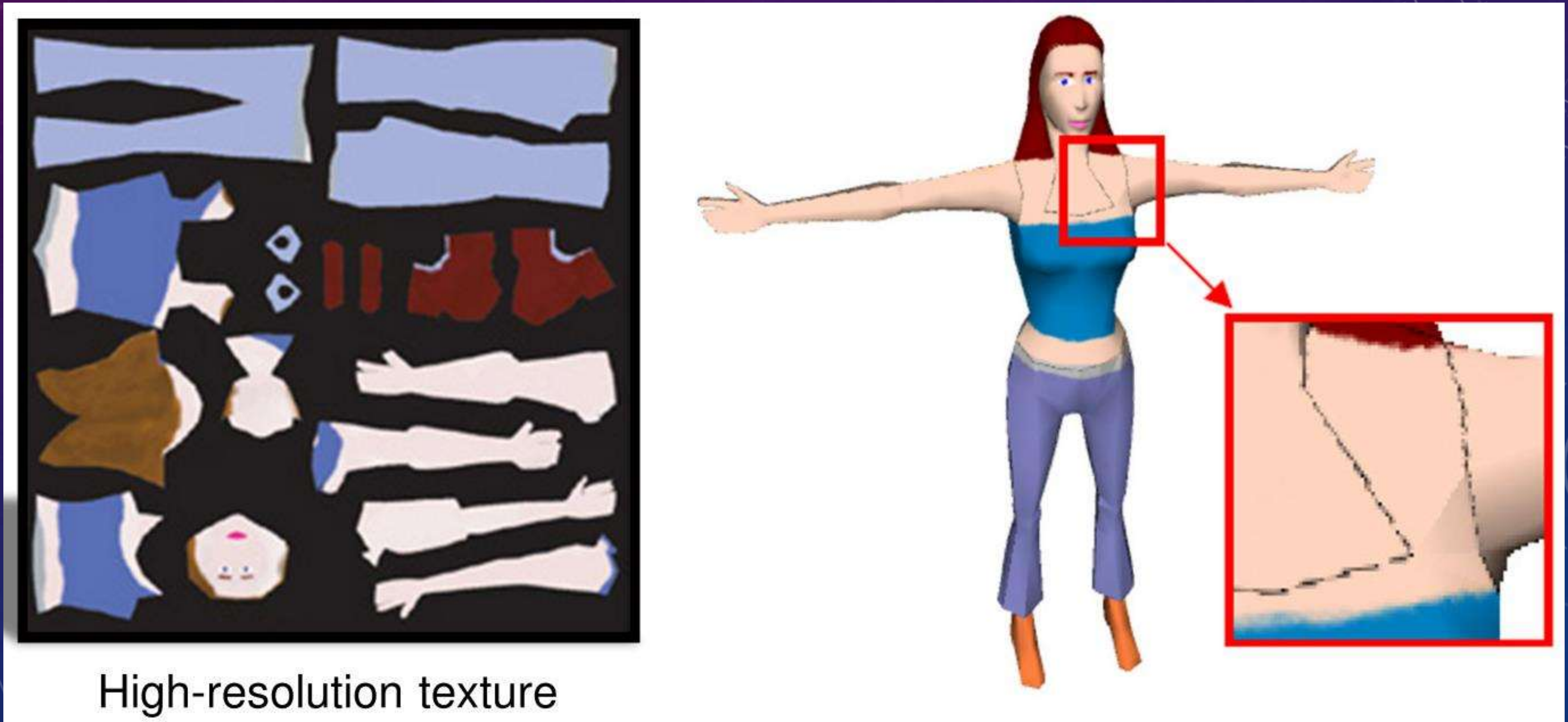
Packing

Mesh Cutting

- Low distortion
- As short as possible length

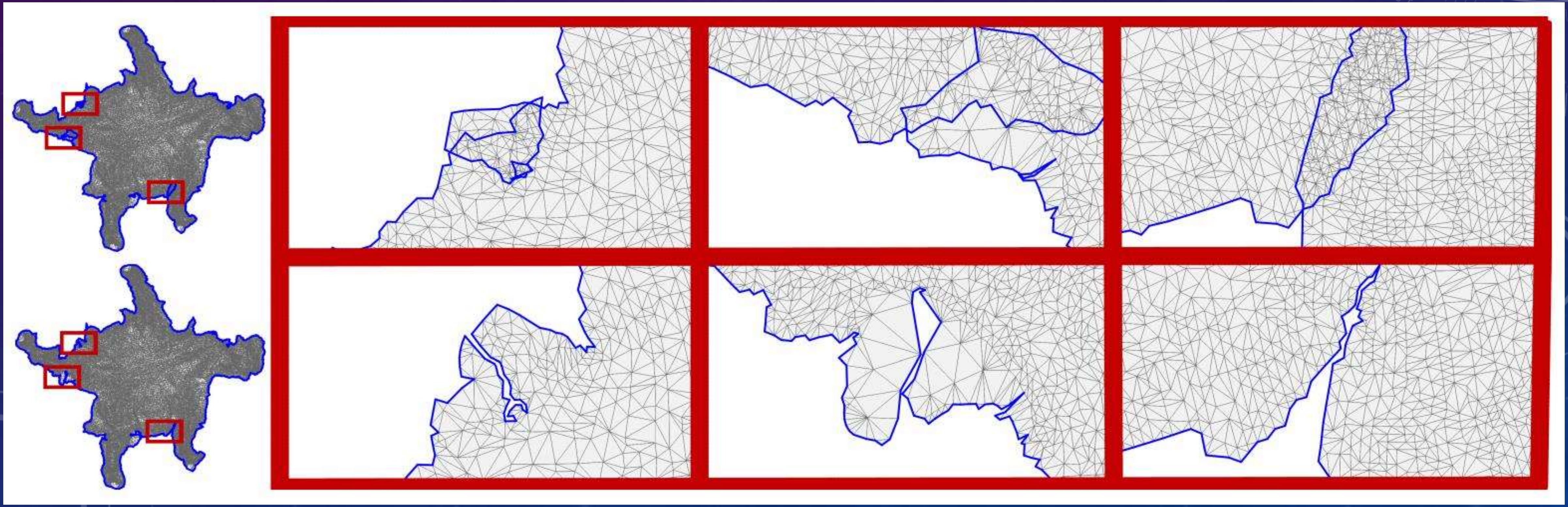


Seams introduce filtering artifacts

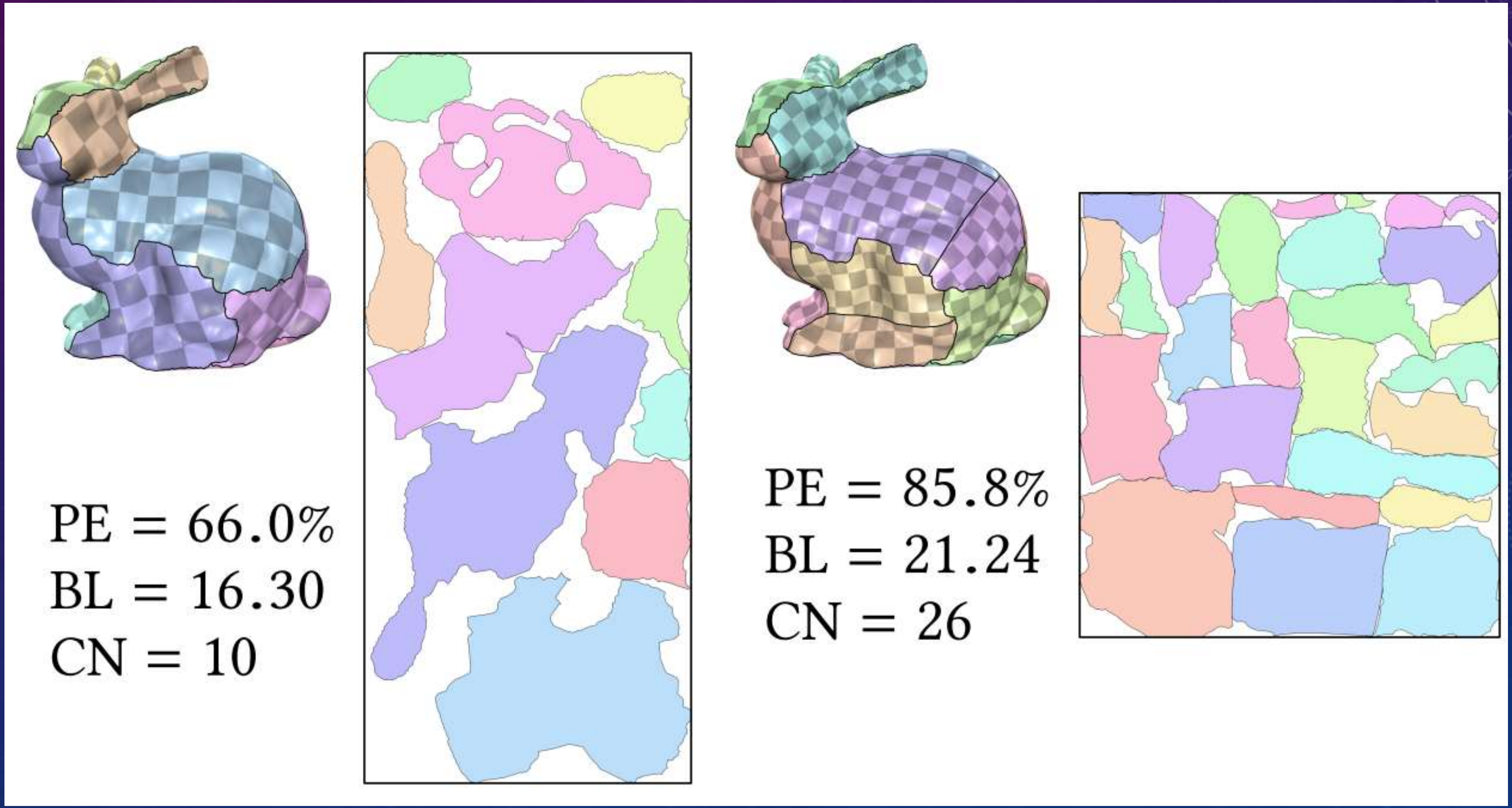


Parameterizations

- Bijective and low isometric distortion

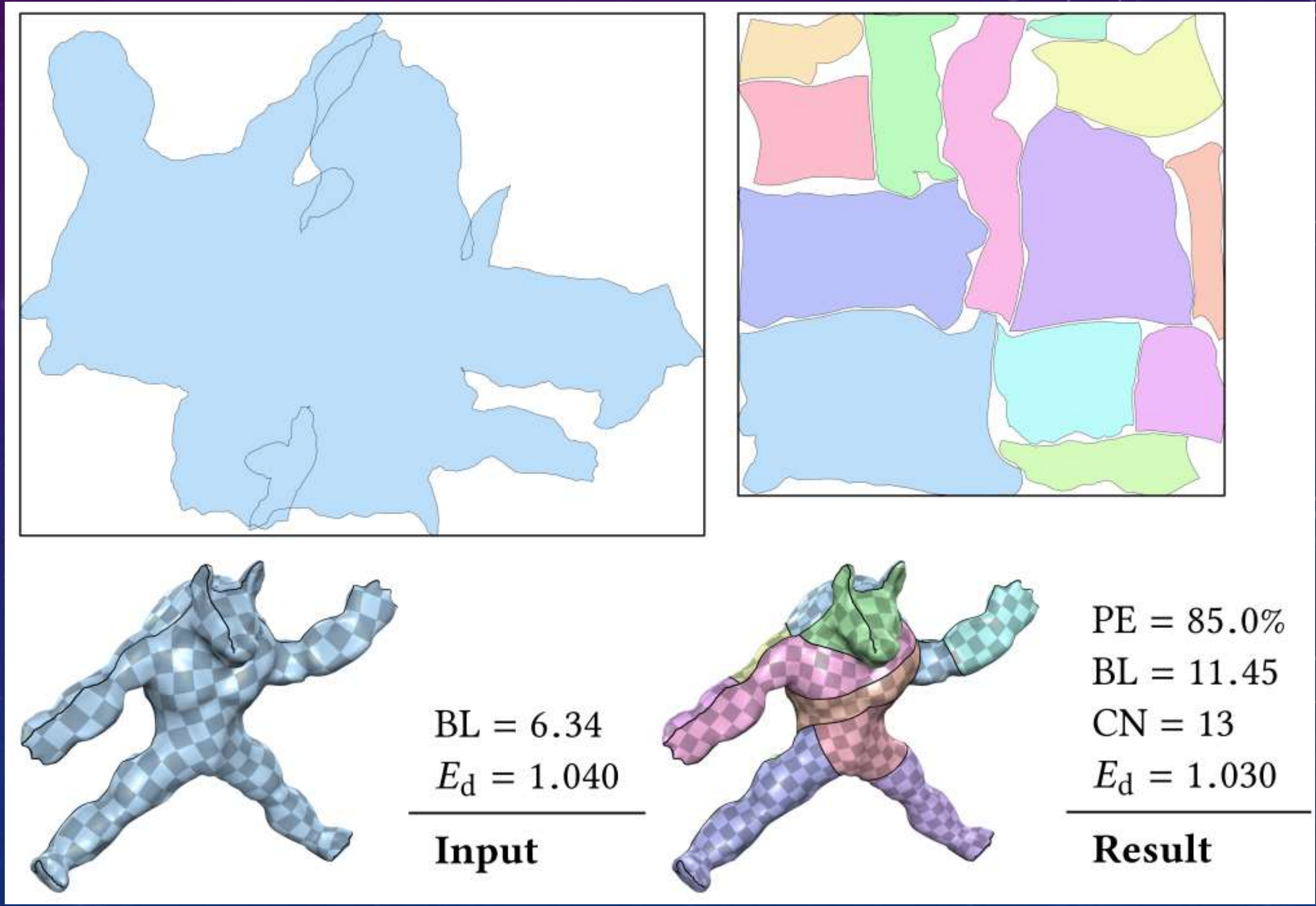


Packing - high packing efficiency



Packing - high packing efficiency

- Mesh cutting
- Parameterization
- Packing

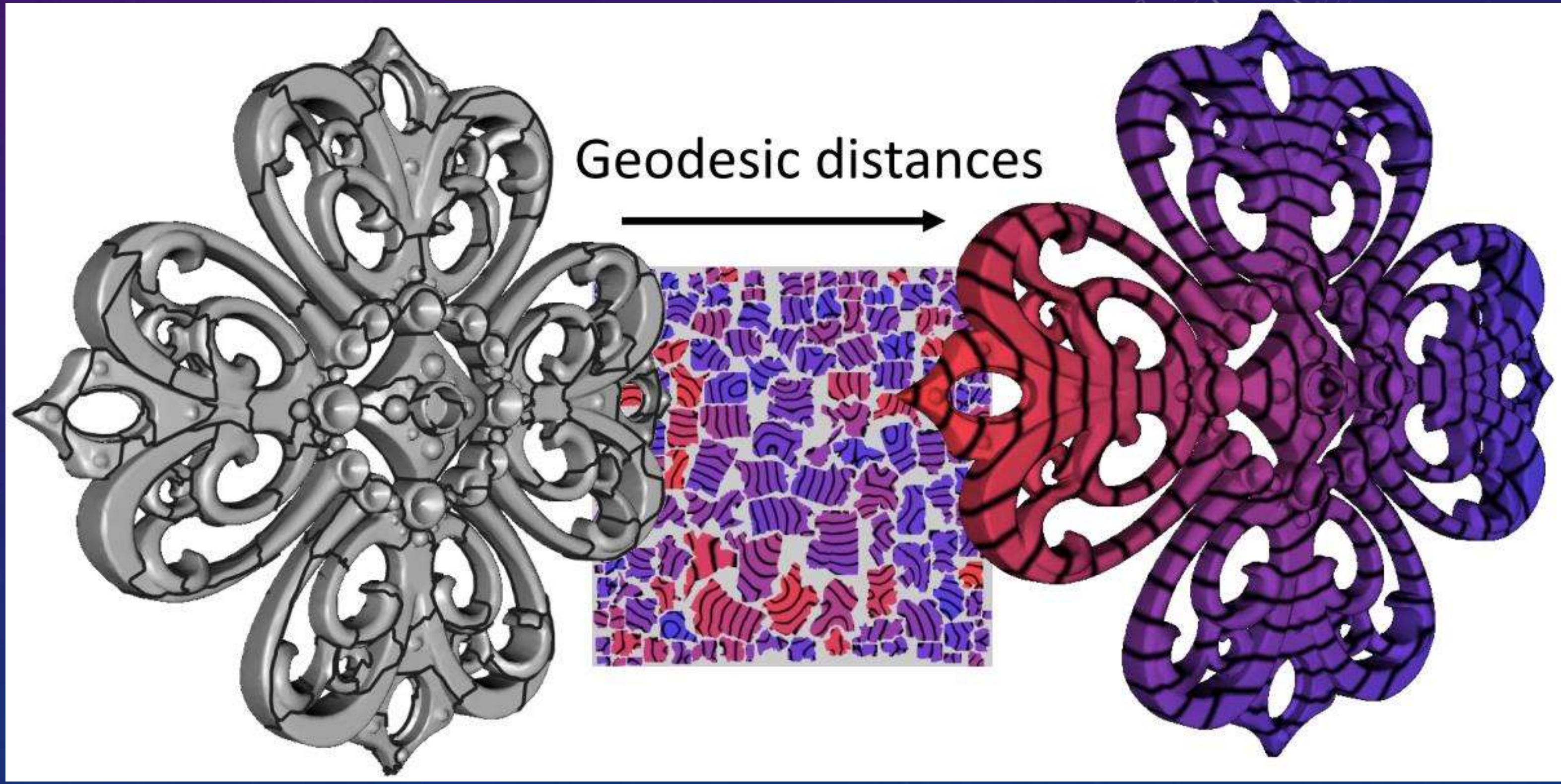


Applications

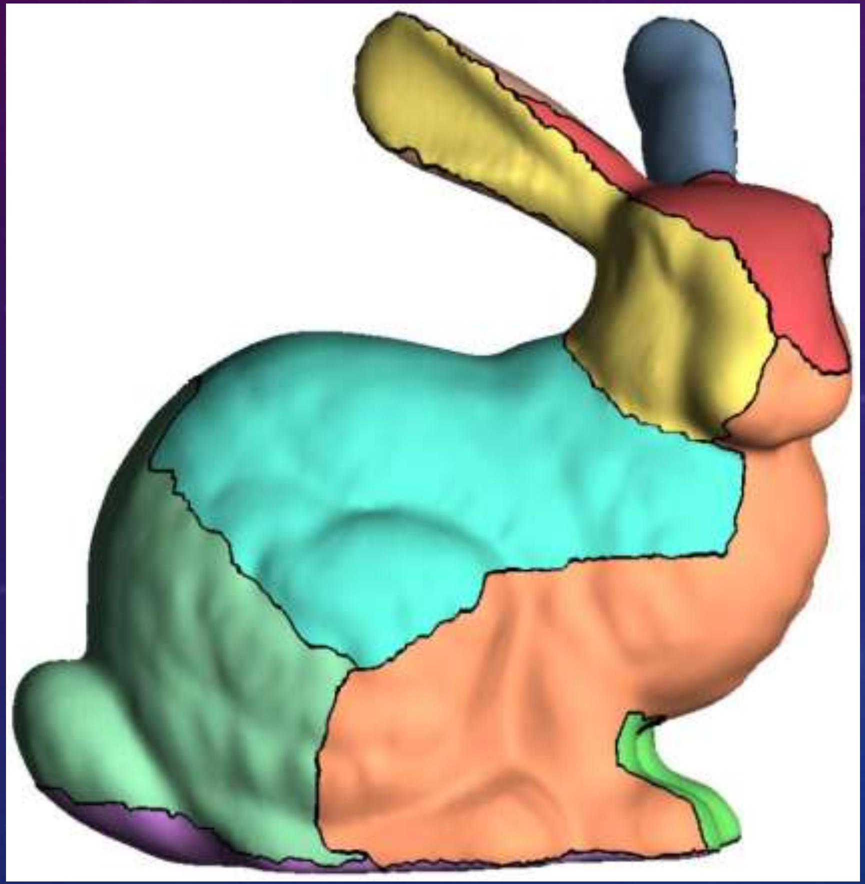
- Signal storage
- Geometric processing

Gradient-domain processing

within a Texture Atlas



Mesh cutting

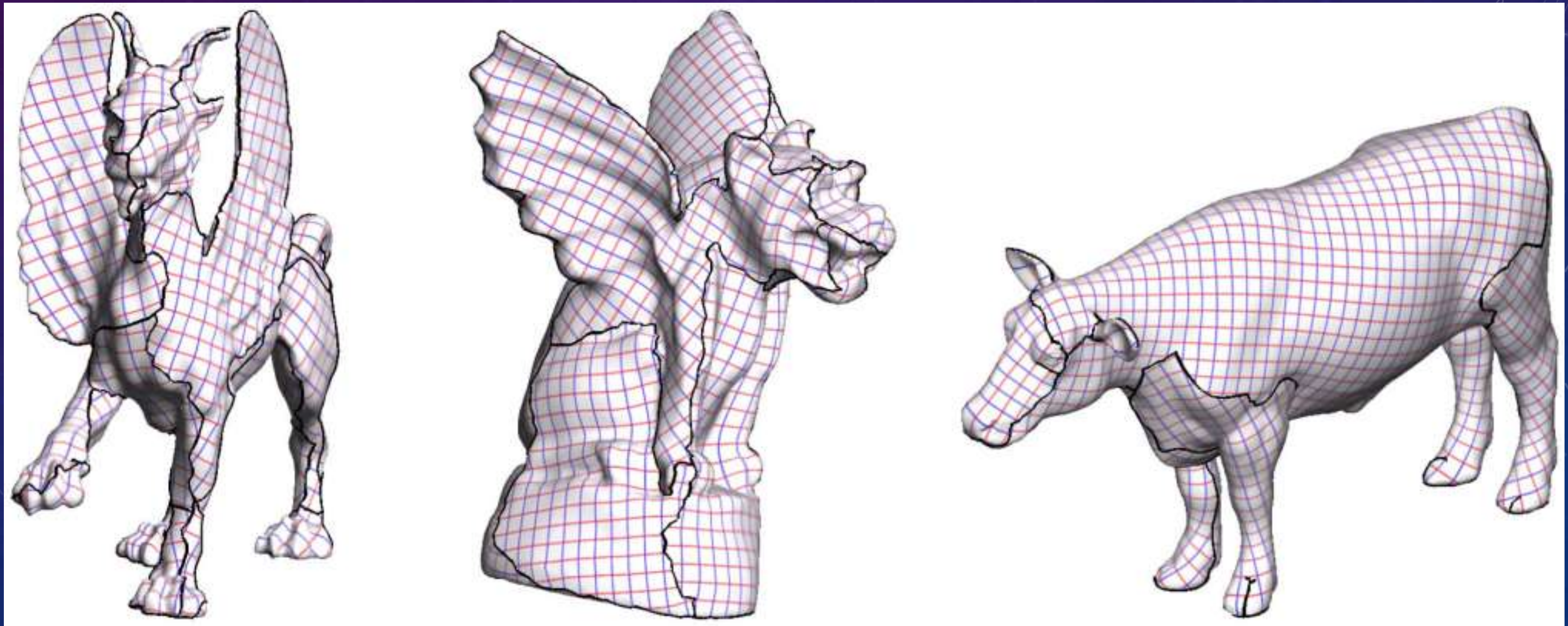


Segmentation

Points → Paths

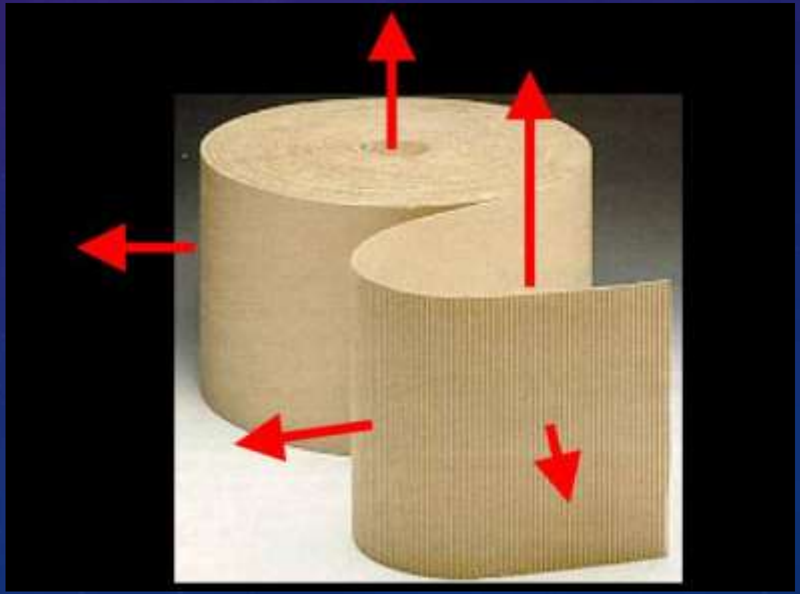
Segmentation

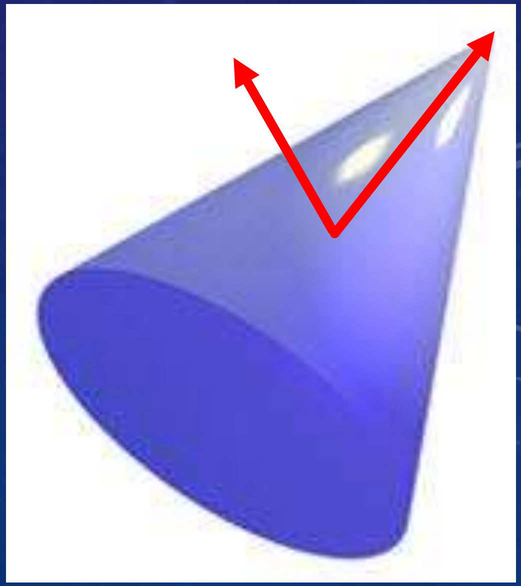
- Goal: mesh segmentation into compact charts with minimal distortion



Proxy

- Developable surfaces of constant slope
- Constant angle between surface normal and axis
- Proxy: $\langle N_c, \theta_c \rangle$





Fitting error

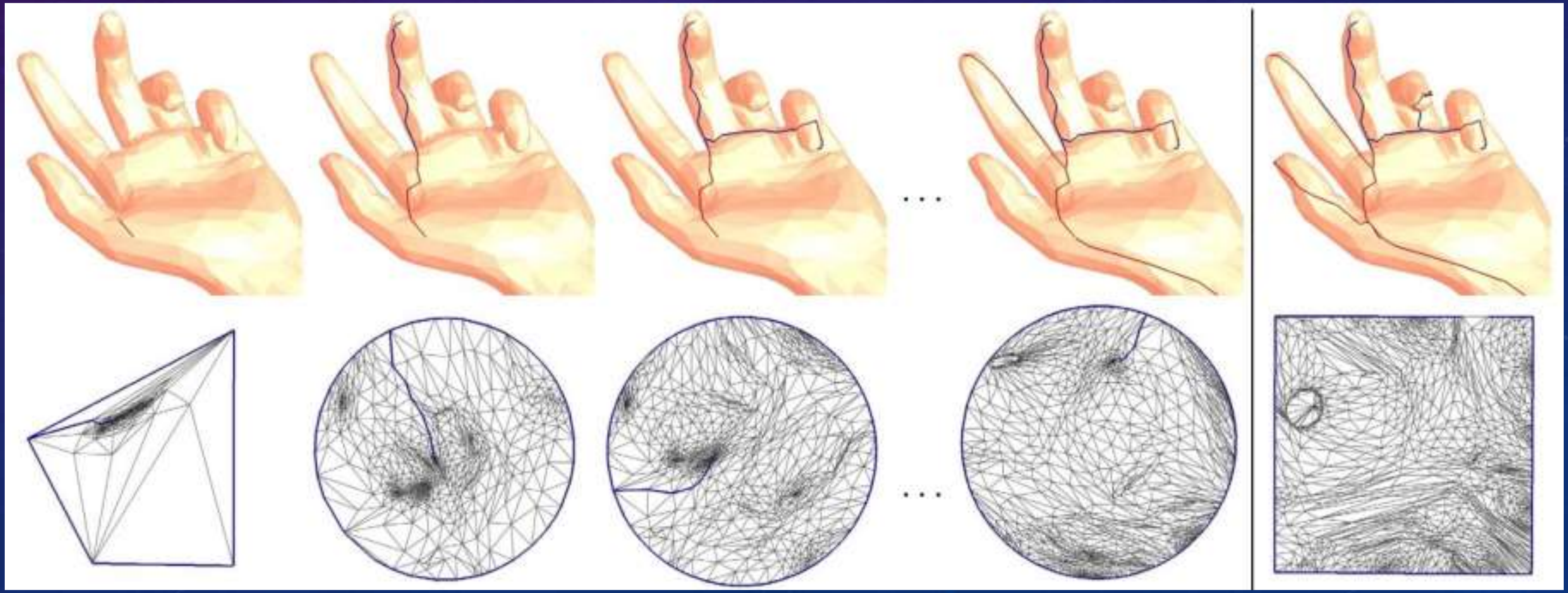
- Measures how well triangle fits a chart $\mathcal{F}(C,t) = (N_c^T n_t - \cos\theta_c)^2$
- Compactness function : $\mathcal{C}(C,t) = \frac{\pi D(S_c,t)^2}{A_c}$
  - S_c is the seed triangle of the given chart
  - $D(S_c,t)$ is the length of the shortest path (inside the chart) between two triangles
  - A_c is the area of chart C
- Cost energy : $E(C,t) = A_t \mathcal{F}(C,t)^{\alpha} \mathcal{C}(C,t)^{\beta}$

Segmentation method

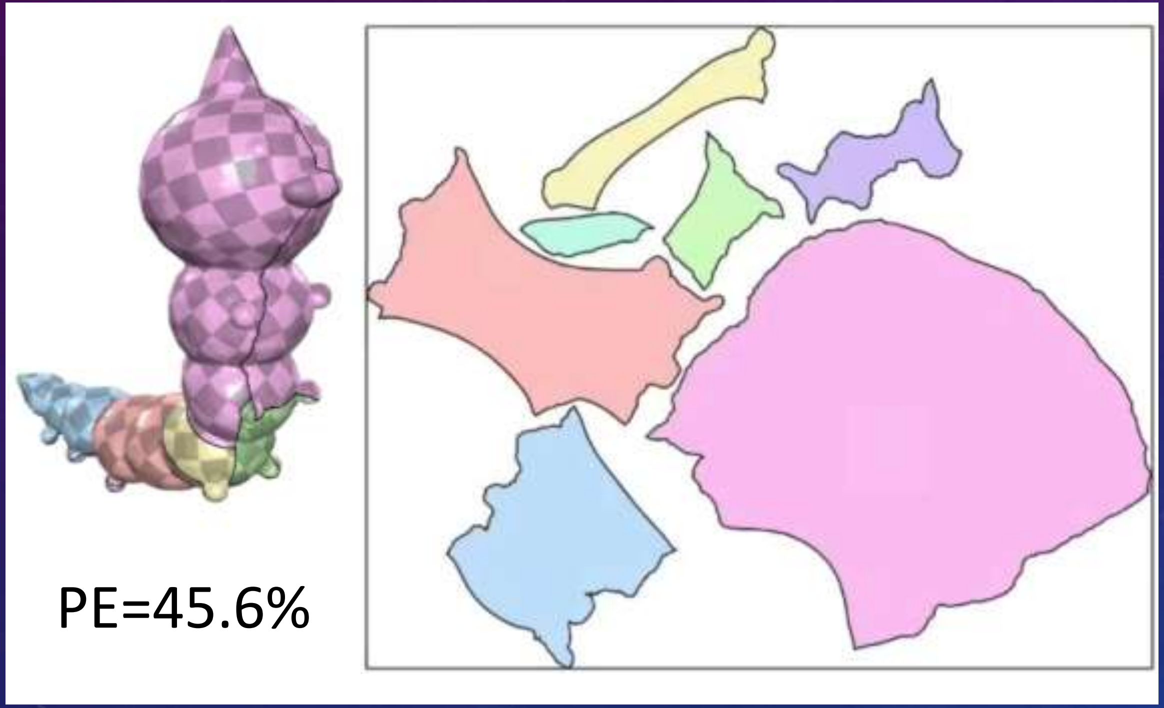
- Lloyd algorithm
  1. Select random triangles to act as seeds
  2. Grow charts around seeds using a greedy approach
  3. Find new proxy for each chart
  4. Repeat from step 2 until convergence
- K-means
- CVT

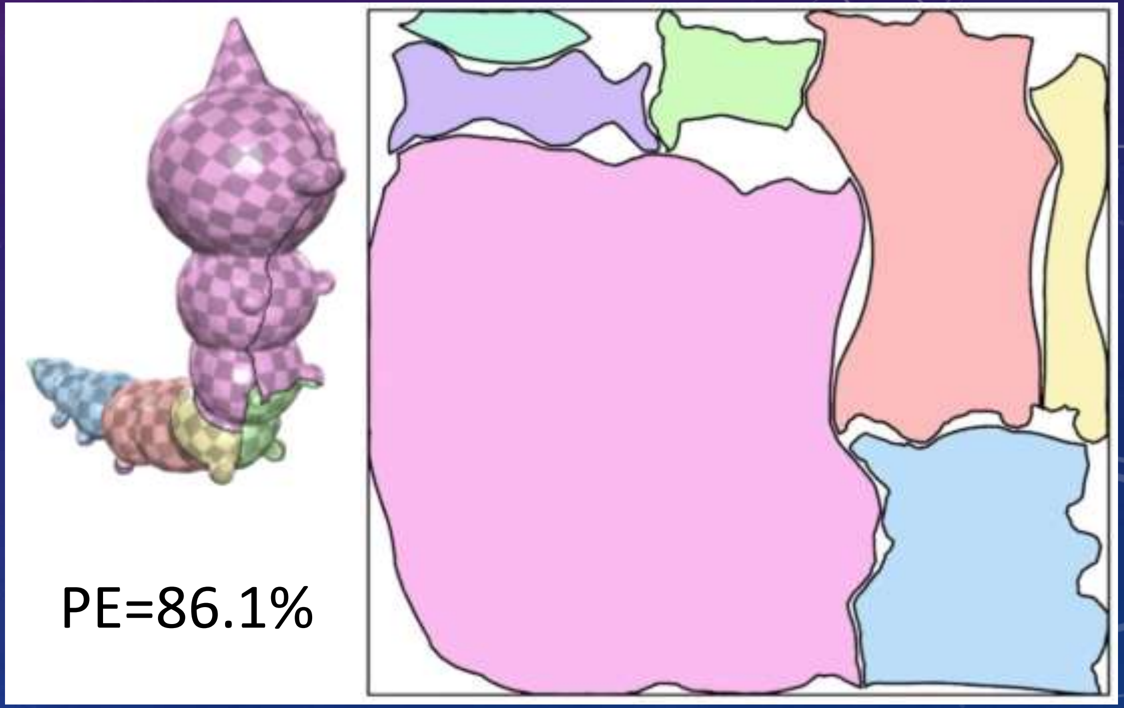
Distortion points - iterative method

- Parameterize the mesh to the plane.
- Add the point of greatest isometric distortion.



Packing efficiency (PE)



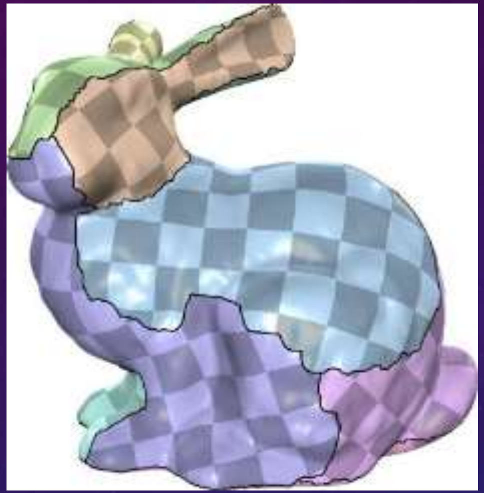


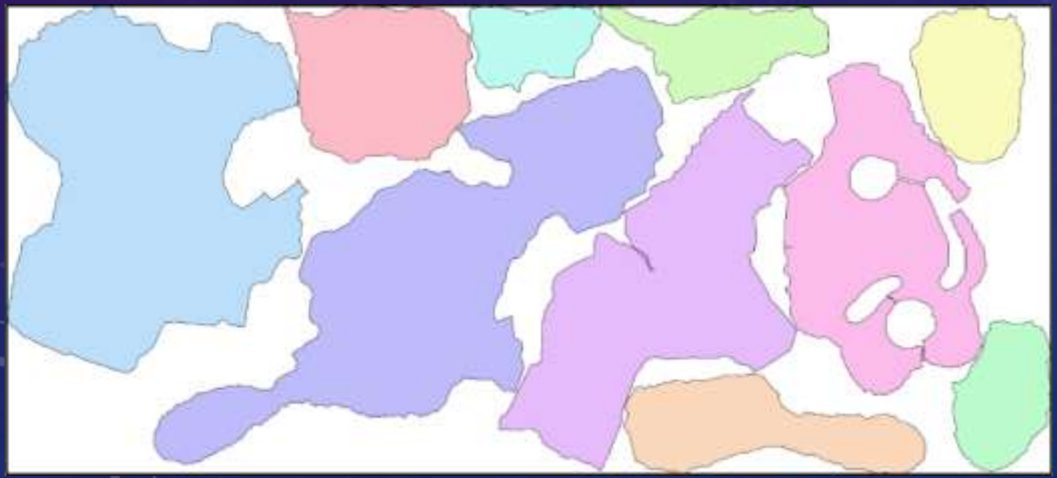
Maximizing atlas packing efficiency is NP-hard!

Other requirements

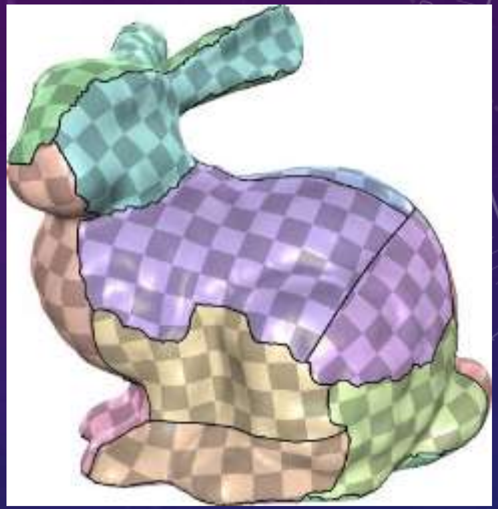
- Low distortion
- Consistent orientation
- Overlap free
- Low boundary length

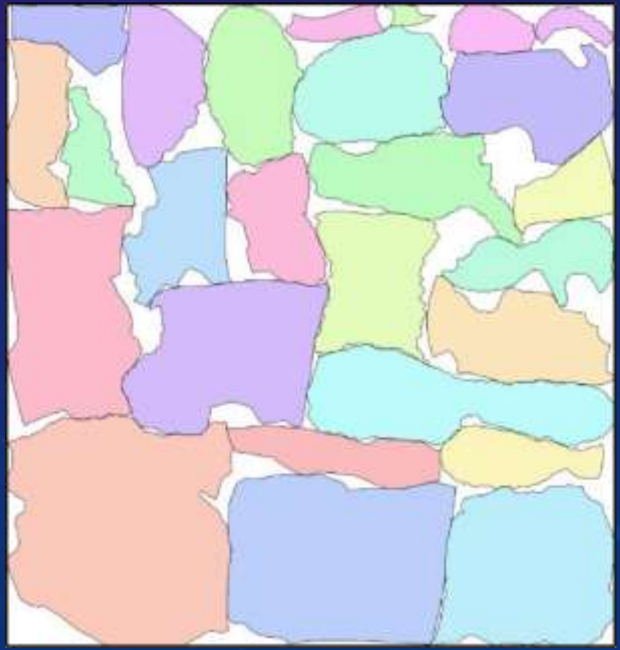
Atlas refinement





Input

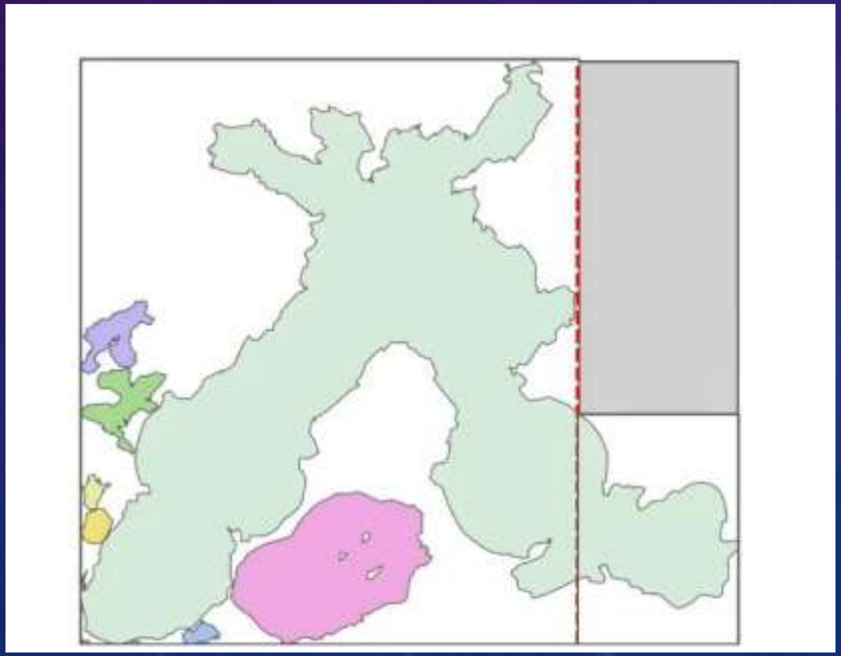


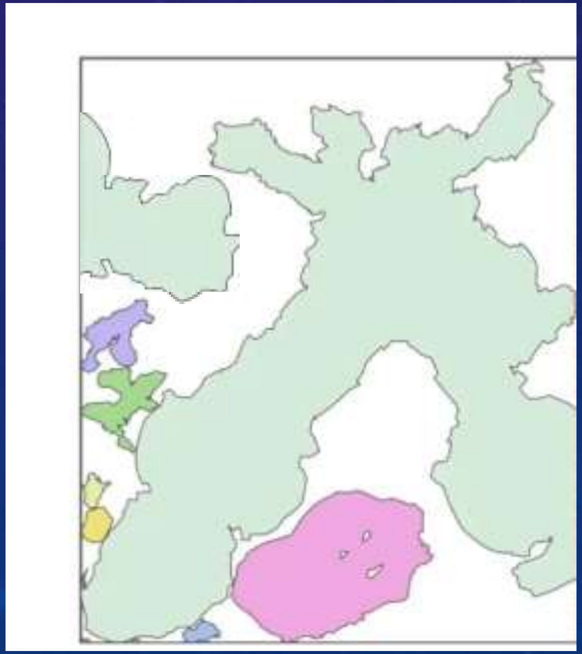


No overlap
High PE

Box Cutter

- Limper, M., Vining, N., & Sheffer, A. (2018). Box cutter: atlas refinement for efficient packing via void elimination. ACM Trans. Graph., 37(4), 153.

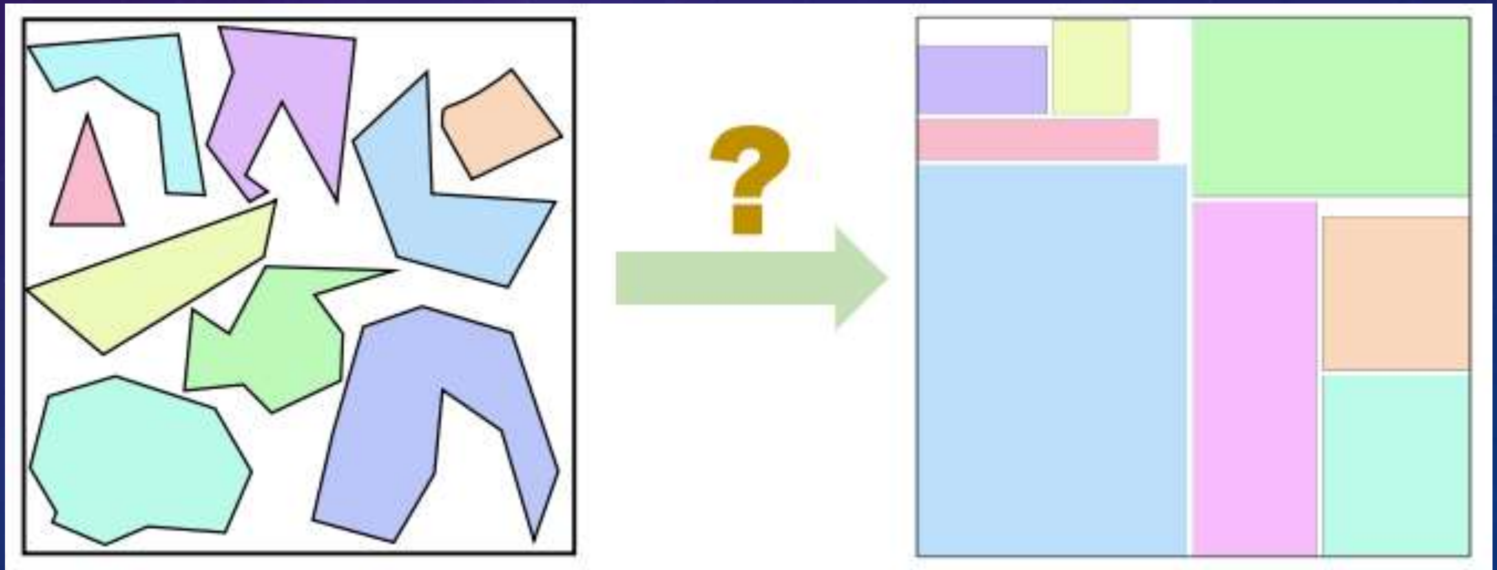




PolyAtlas

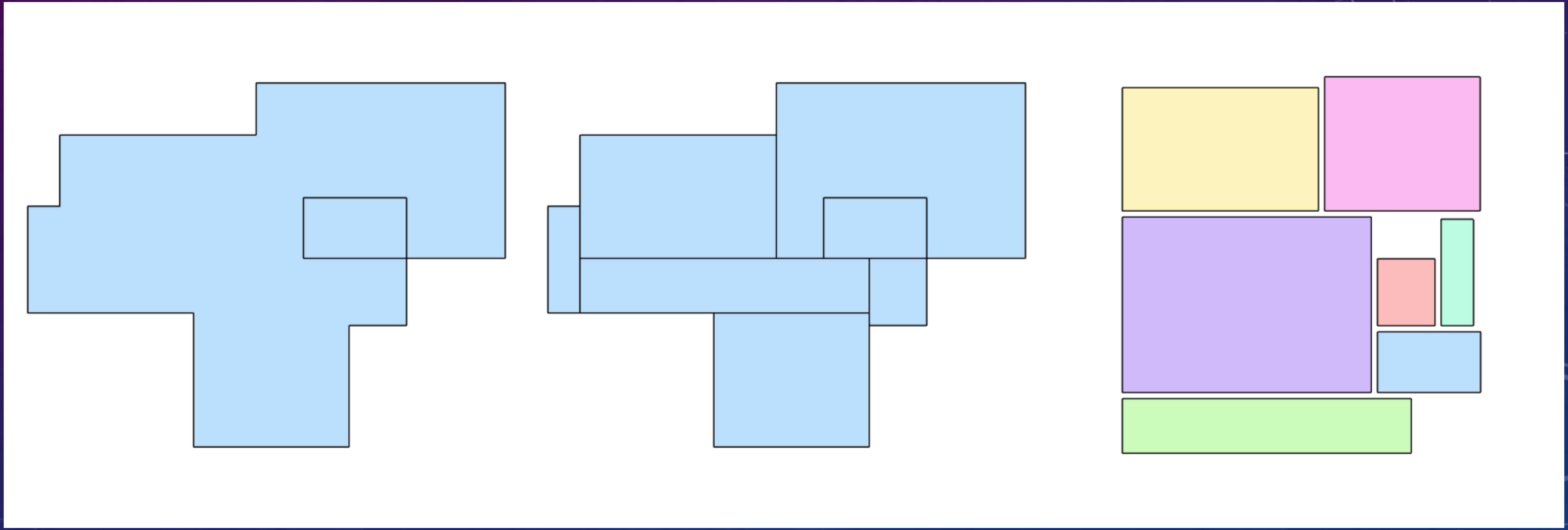
- Liu, H. Y., Fu, X. M., Ye, C., Chai, S., & Liu, L. (2019). Atlas refinement with bounded packing efficiency. ACM Transactions on Graphics (TOG), 38(4), 1-13.

Irregular shapes
Hard to achieve high PE



Rectangles
Simple to achieve high PE

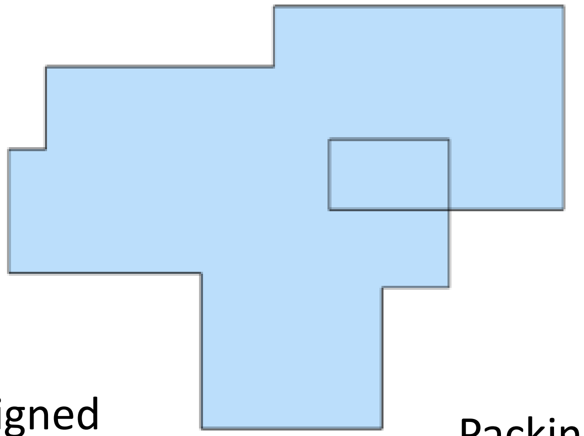
Axis-aligned structure



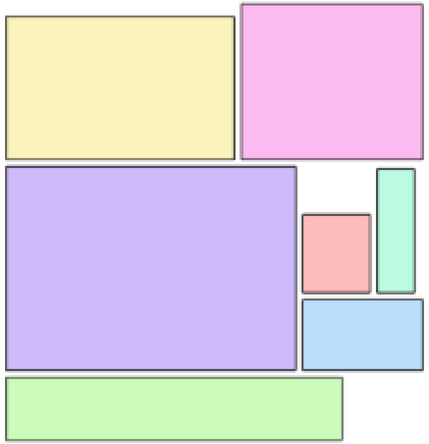
Axis-aligned structure   Rectangle decomposition   High PE (87.6%)!

General Cases

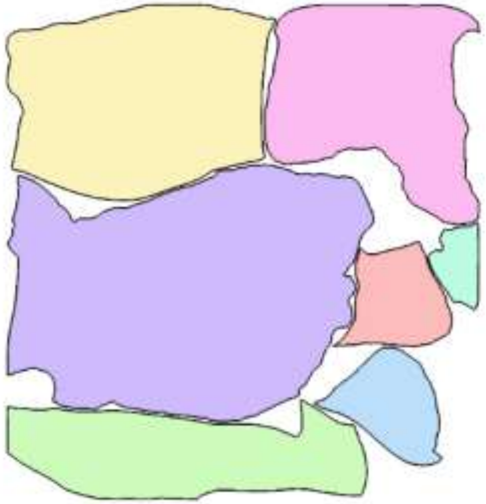
Axis-aligned deformation



Packing



Scaffold



Applications

- Atlas generation
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- Inter-surface mappings

Computational Peeling Art Design

ACM SIGGRAPH 2019

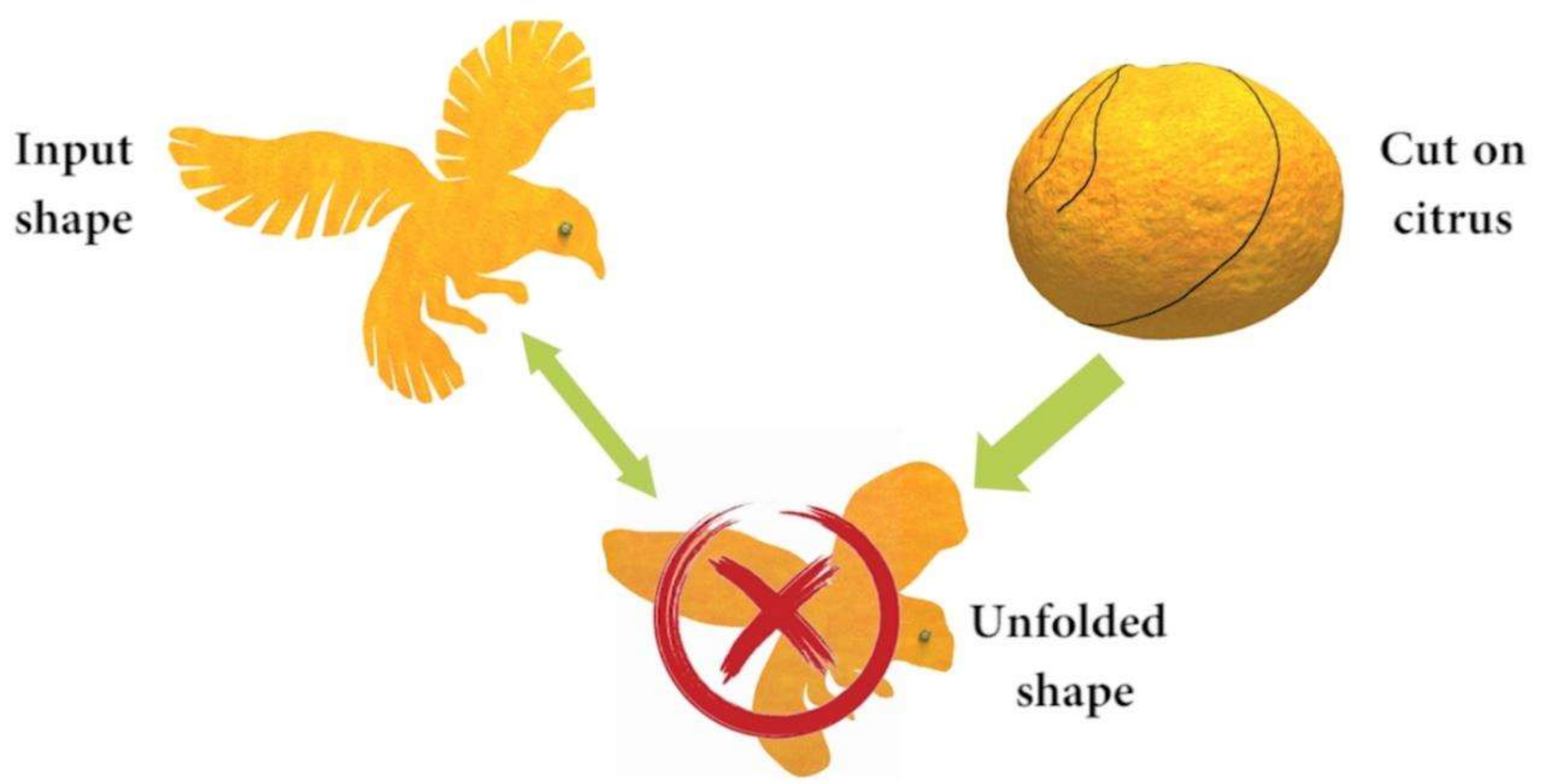
Hao Liu* Xiao-Teng Zhang* Xiao-Ming Fu Zhi-Chao Dong Ligang Liu

University of Science and Technology of China

(This video contains voiceover.)

Peeling art

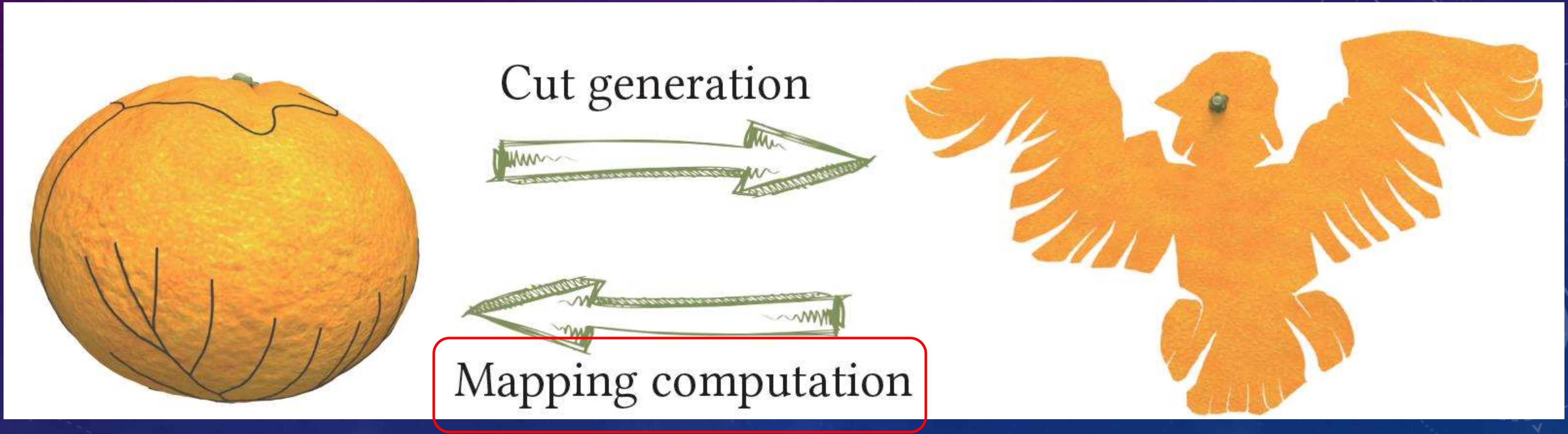
Problem



Problem

- Cut generation
- Shape similarity

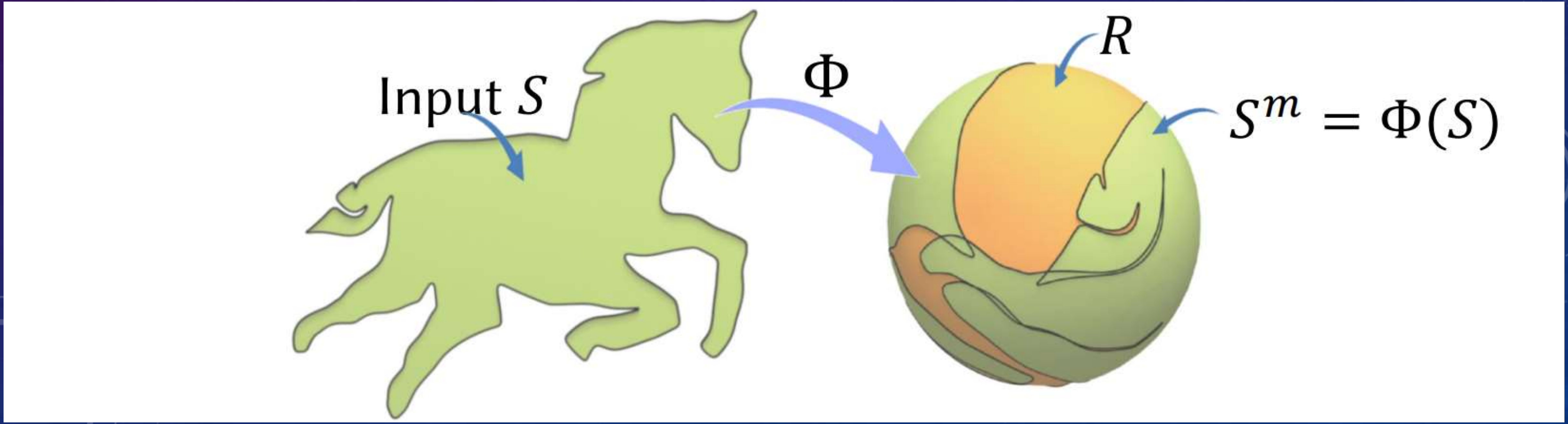
Inverse problem



Inverse problem

- Low isometric distortion for ϕ
- Area of remain regions → 0

$$\min E_{iso}(S^m, S) + wE_{shr}(R)$$



Energy

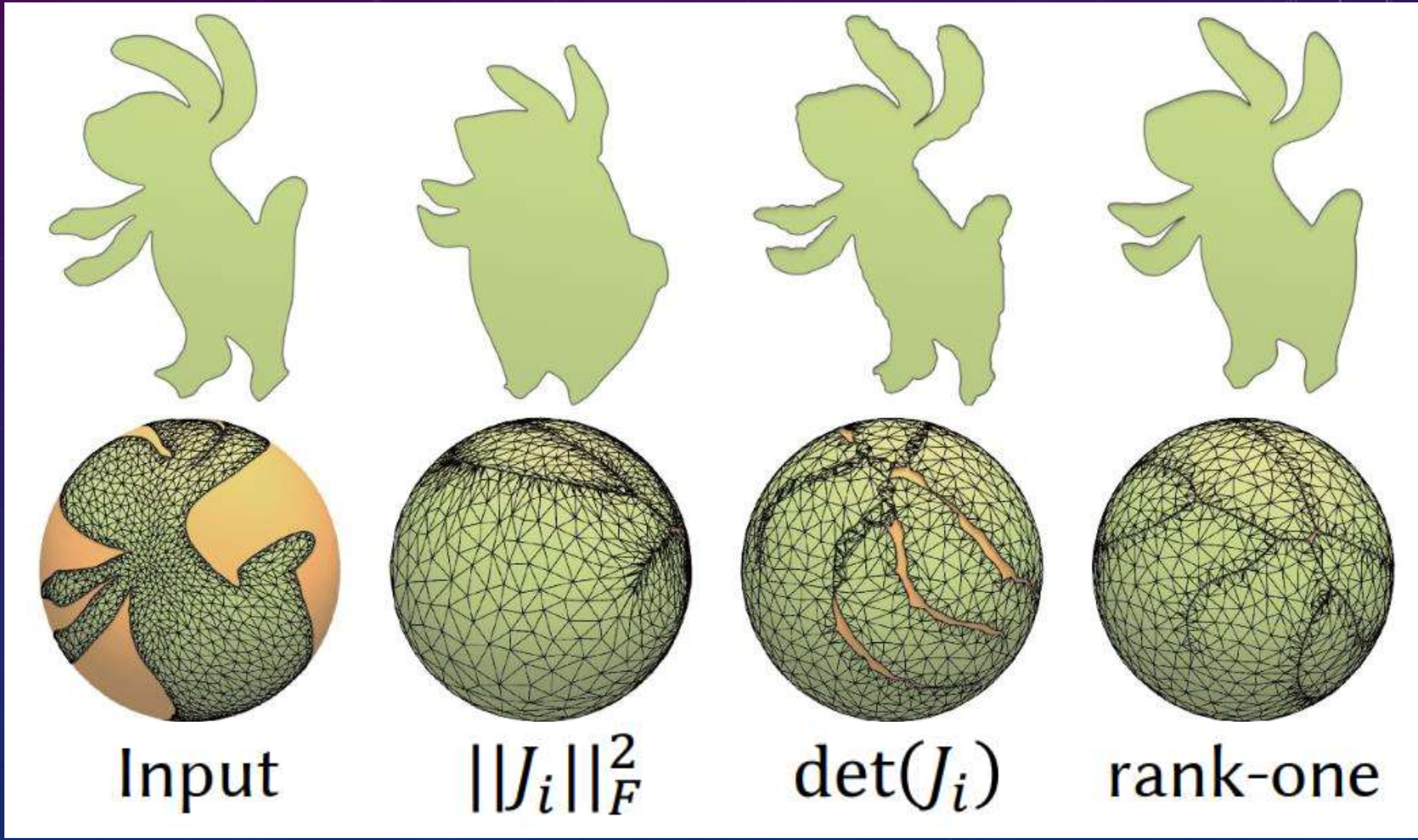
- ARAP distortion metric [Liu et al. 2008]

$$E_{iso}(S^m, S) = \sum_{ijk \in S} A_{ijk} \left\| J_{ijk} - R_{ijk} \right\|_F^2, \qquad R_{ijk} R_{ijk}^T = I$$

- Area shrink energy

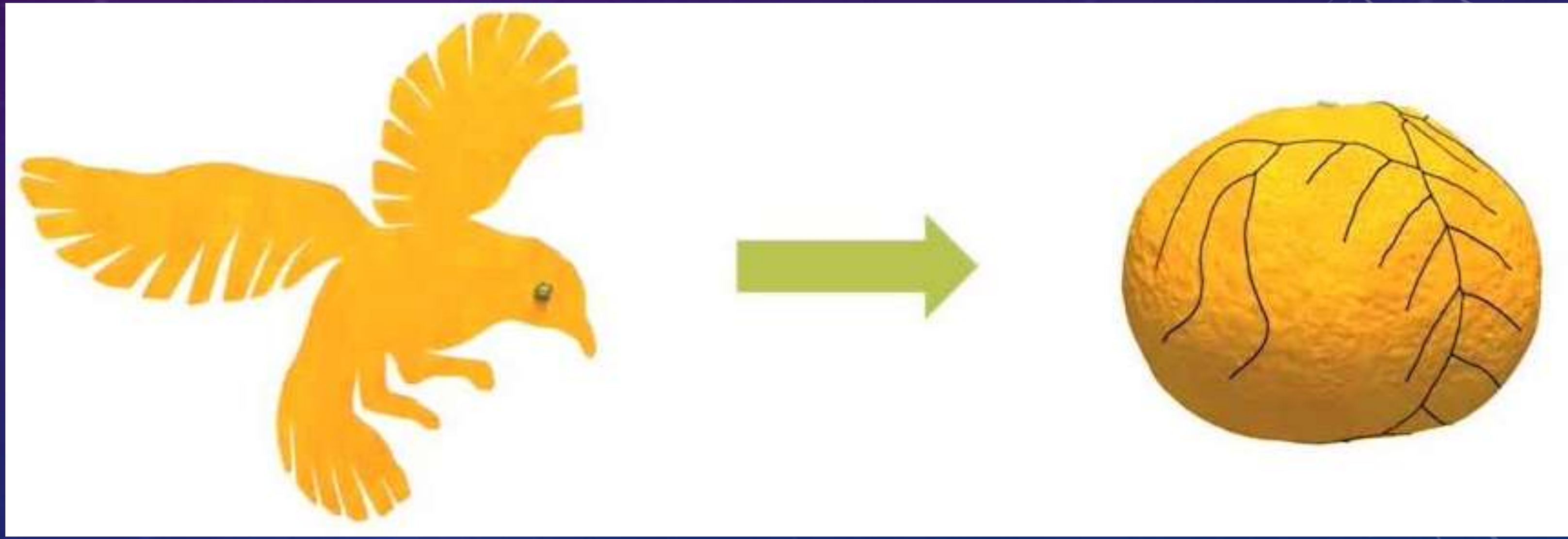
$$E_{shr}(R) = \sum_{ijk \in R} A_{ijk} \left\| J_{ijk} - B_{ijk} \right\|_F^2, \qquad rank(B_{ijk}) = 1$$

Different shrink energy



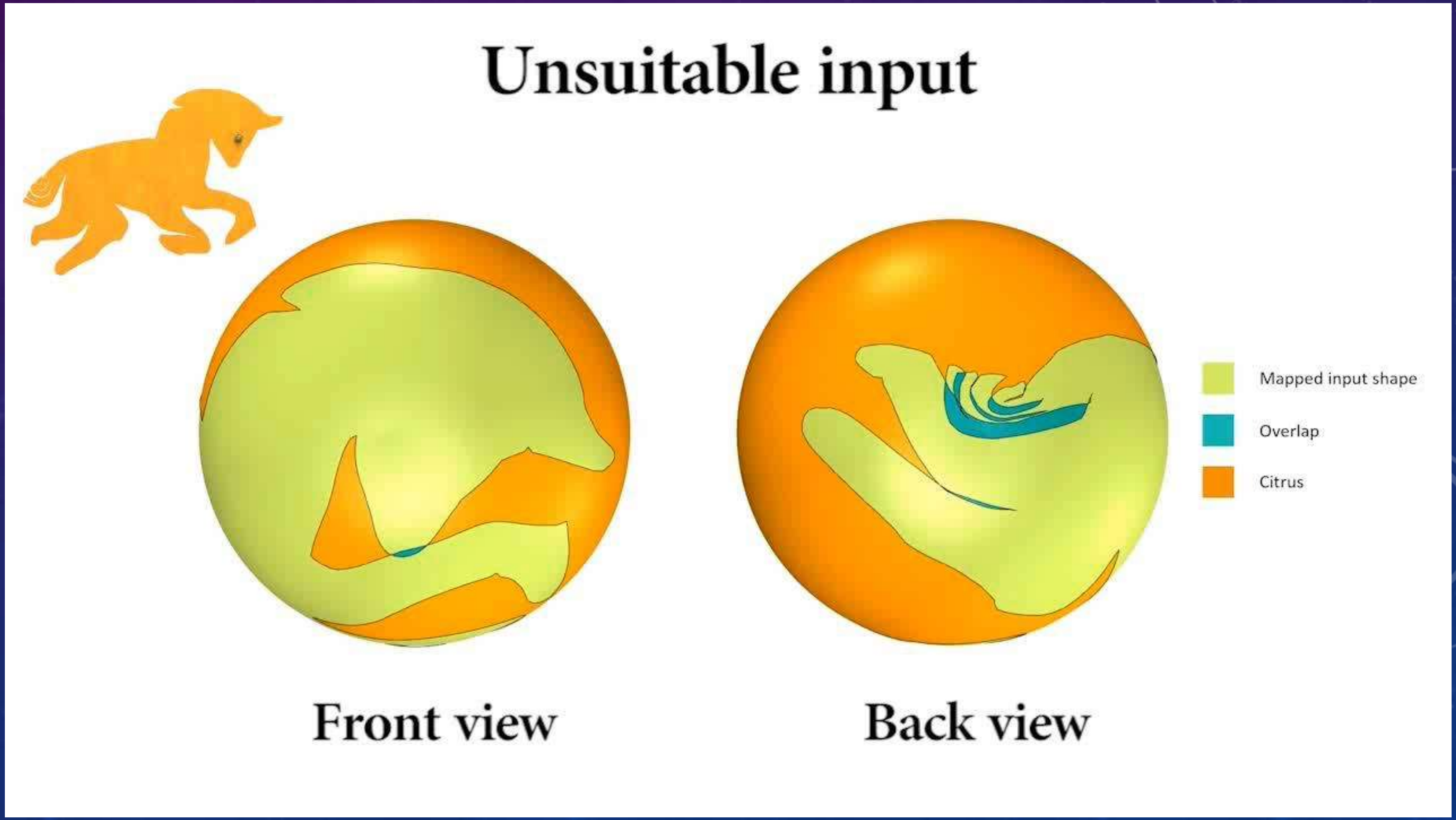
Different initialization

- Suitable input

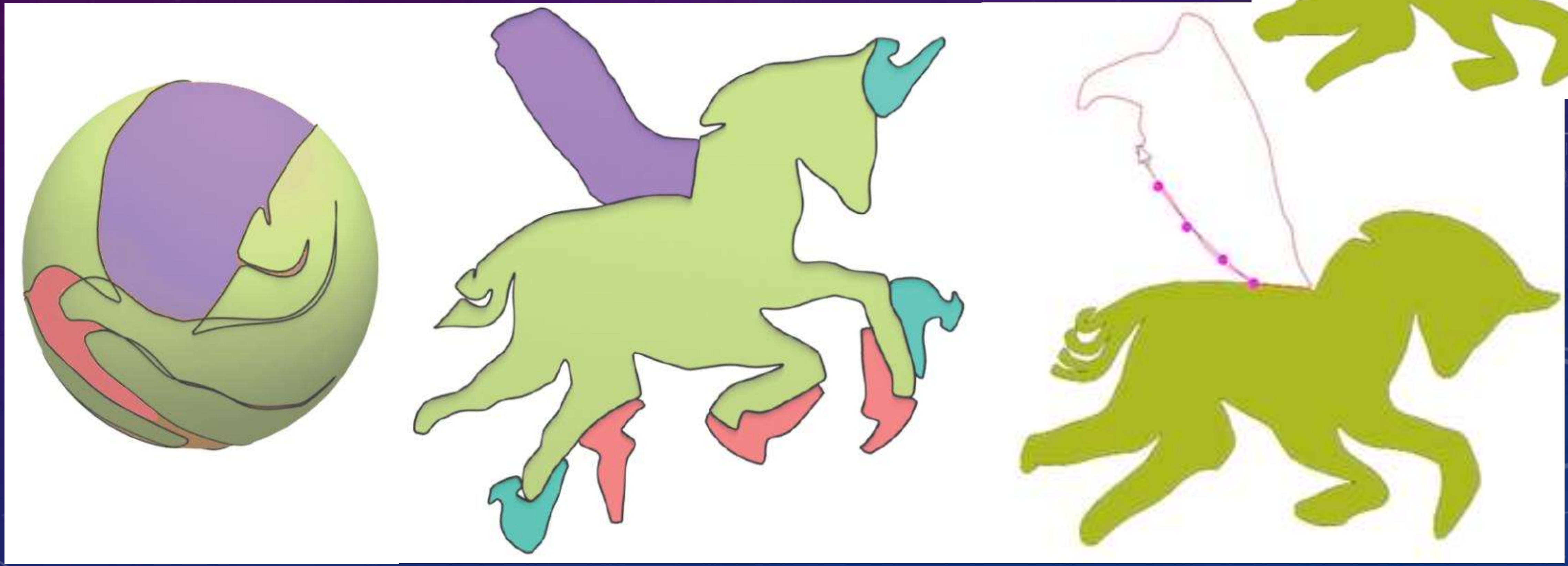


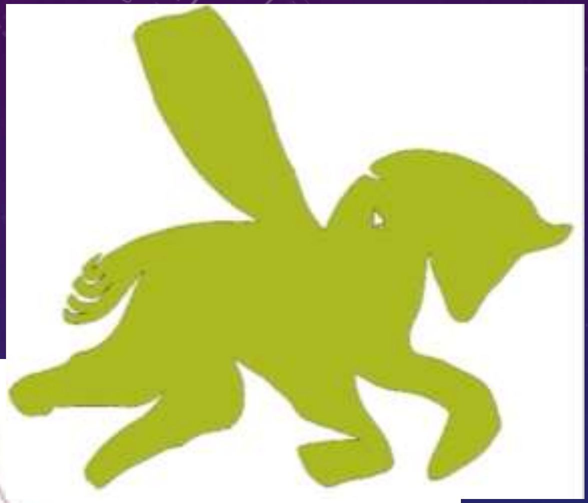
Different initialization

- Suitable input
- Unsuitable input

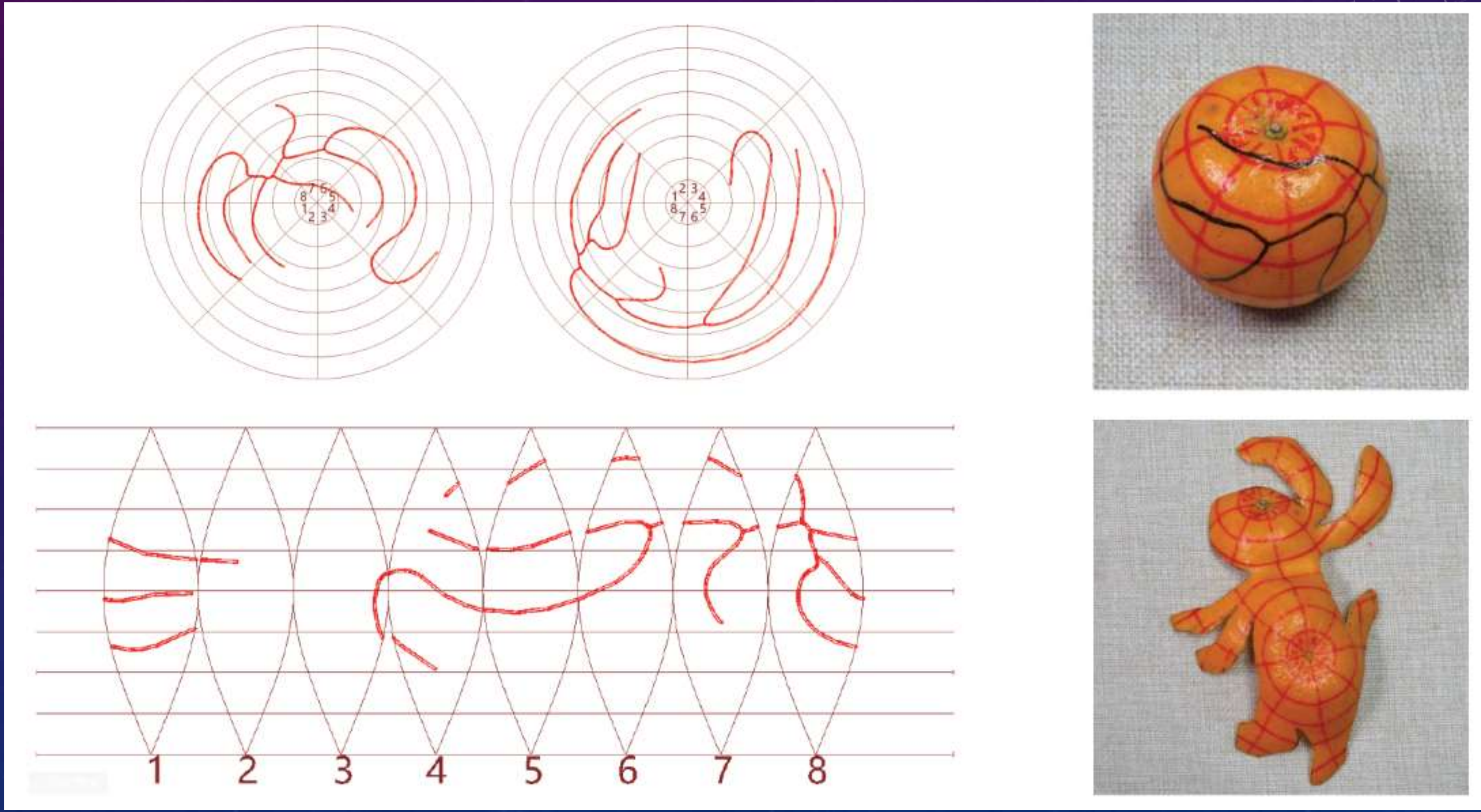


Iterative interaction





Real peeling



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Applications

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- Peeling art
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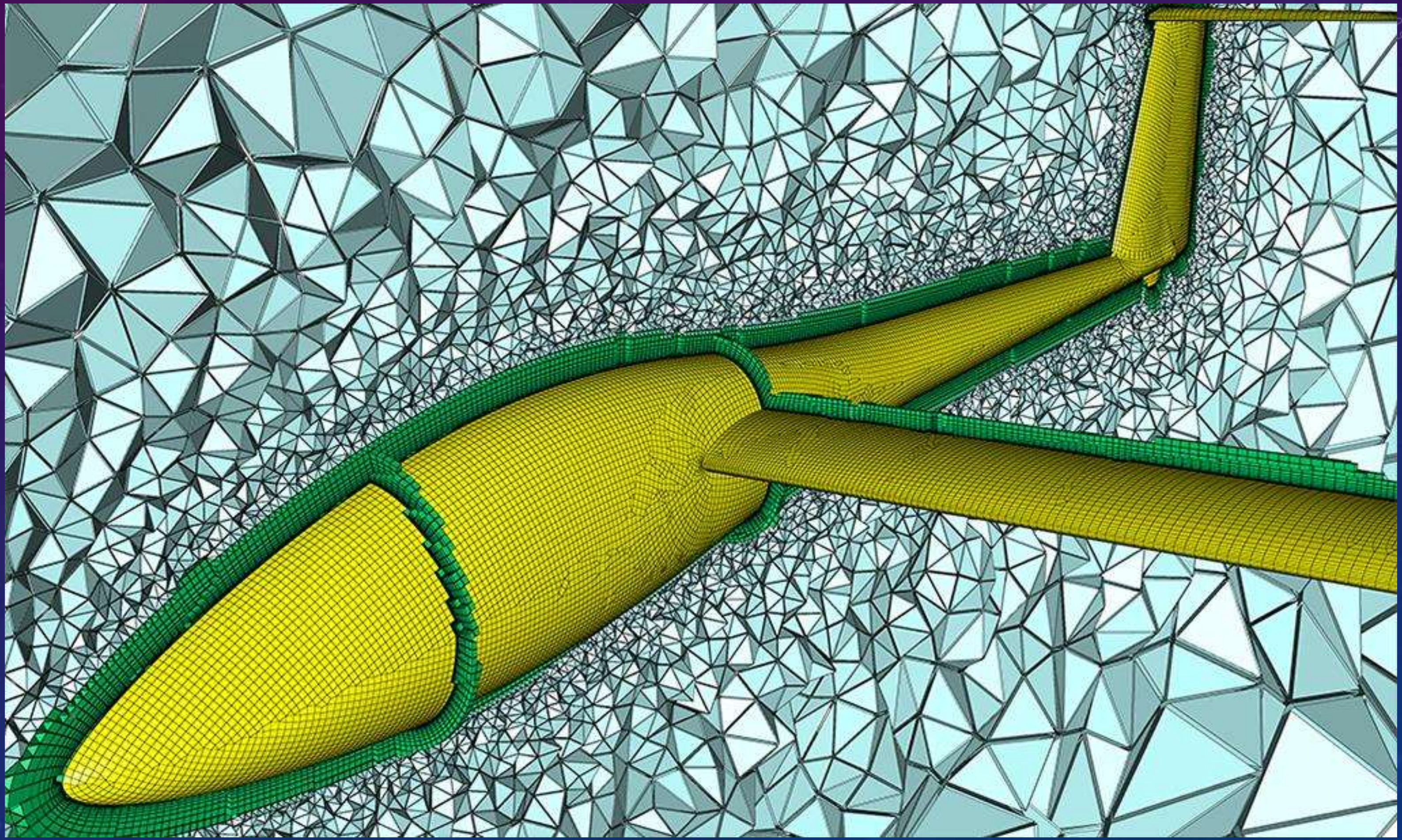
- Inter-surface mappings

Meshing

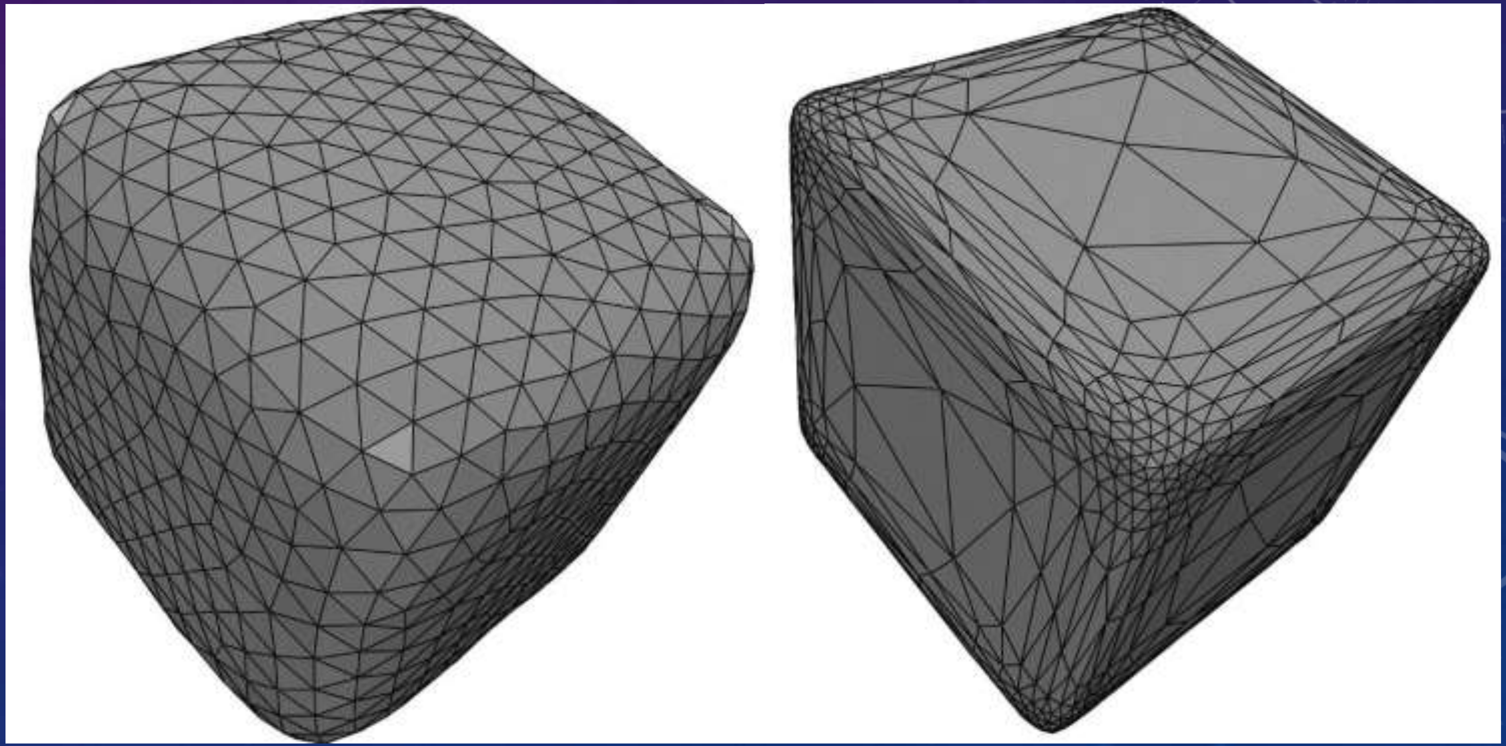


Remeshing

- Given a 3D mesh, compute another mesh, whose elements satisfy some quality requirements, while approximating the input acceptably.
- Mesh quality : sampling density, regularity, size, orientation, alignment, shape of the mesh elements, non-topological issues (mesh repair)
- Different applications imply different quality criteria and requirements.

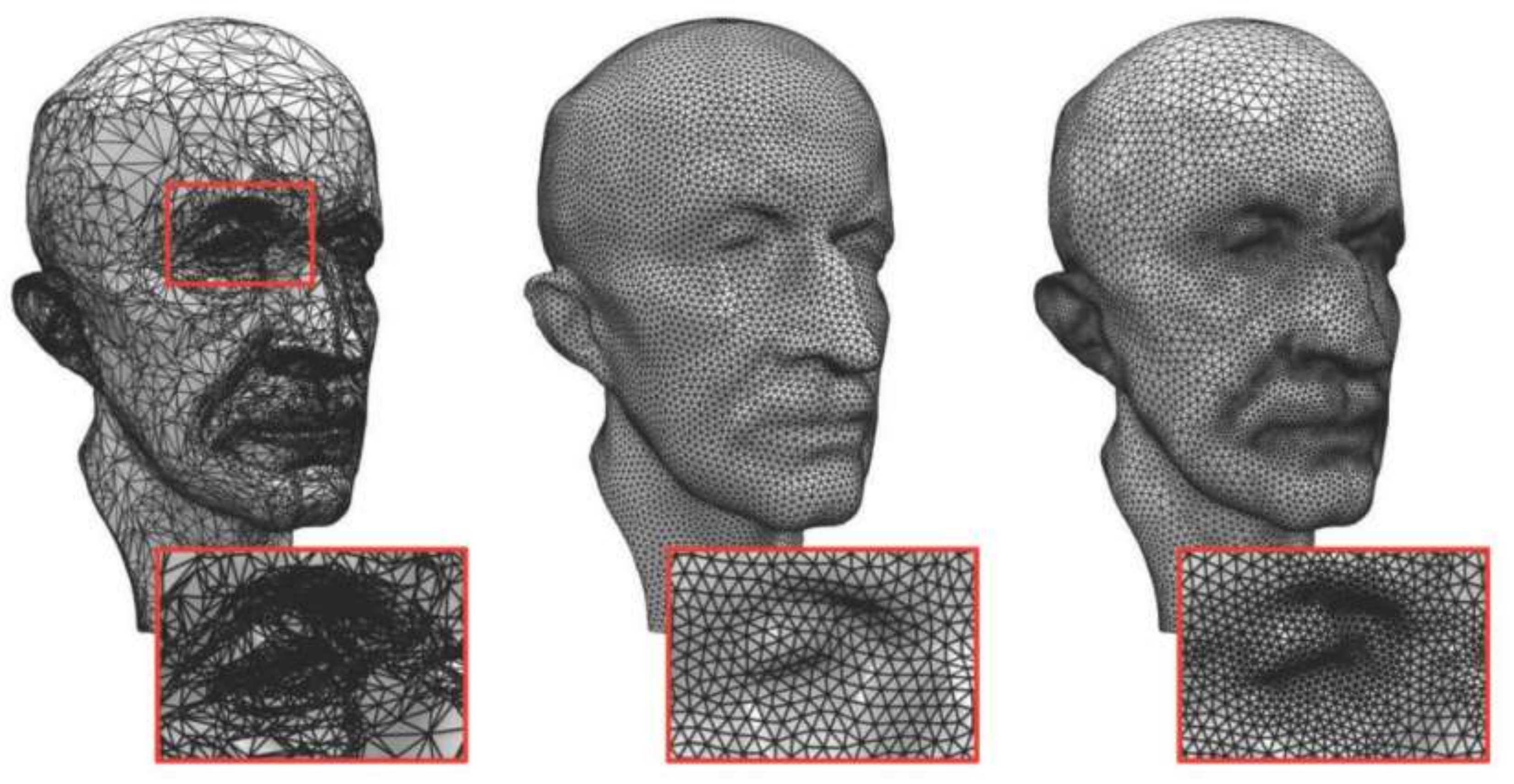
Local structure

- Element shape
  - Isometric
  - Anisotropic



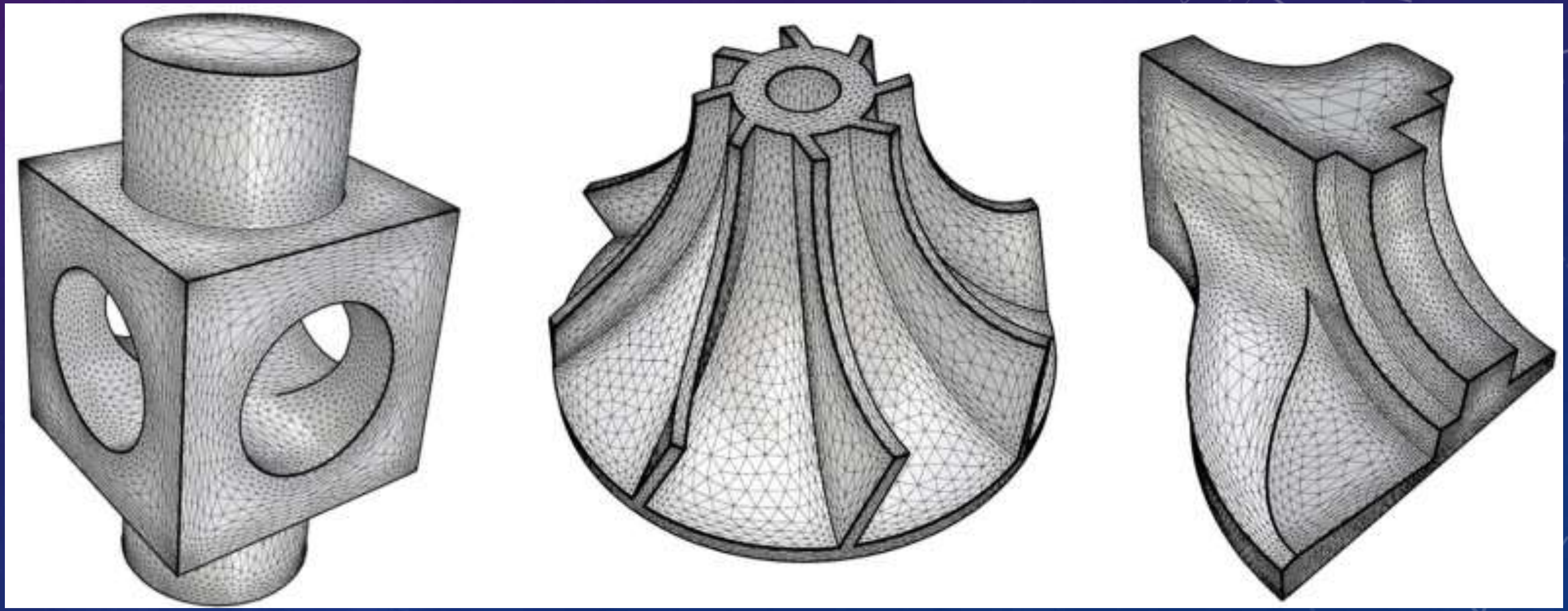
Local structure

- Element shape
- Element density
  - Uniform
  - Adaptive



Local structure

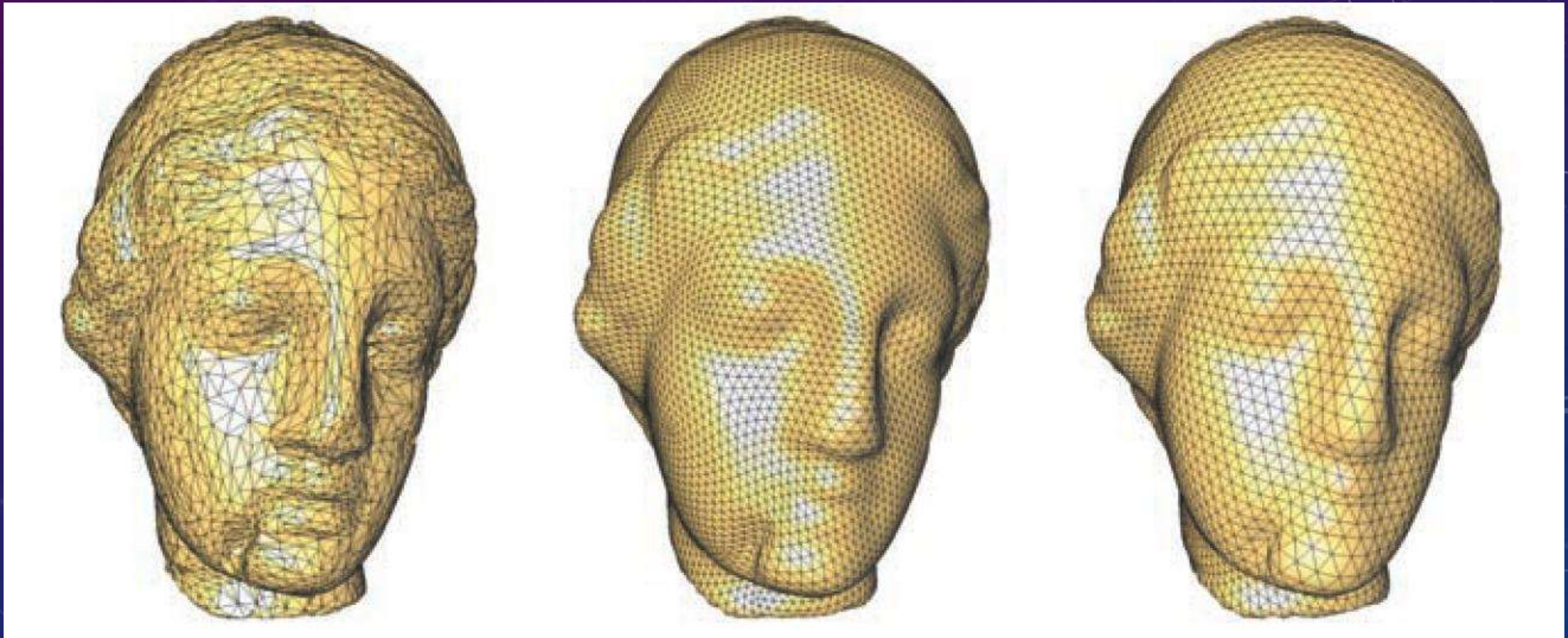
- Element shape
- Element density
- Element alignment
- Anisotropic orientation



Global structure

- Irregular
- Semiregular - regular subdivision of a coarse initial mesh
- Highly regular - most vertices are regular
- Regular - all vertices are regular

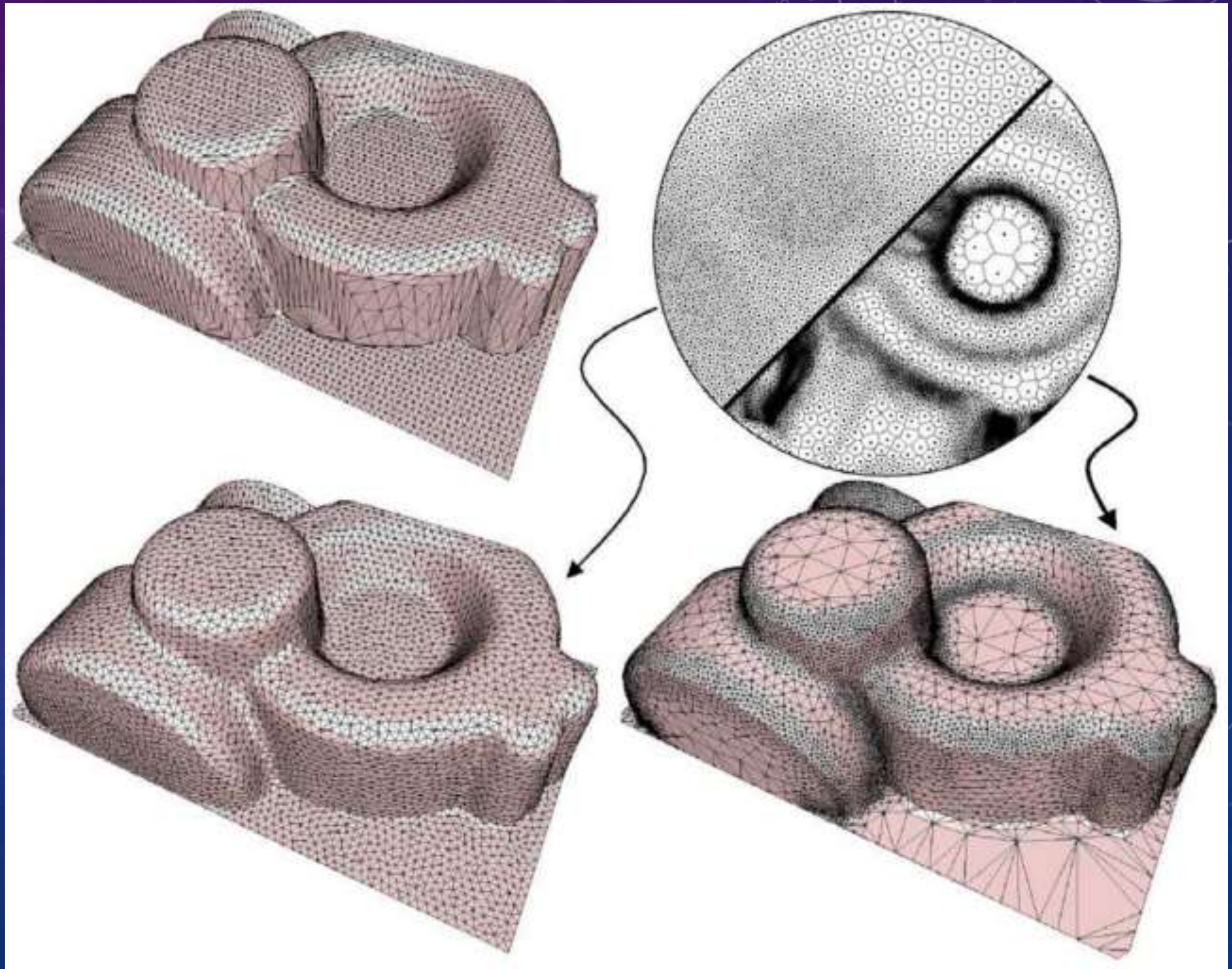
Global structure



Irregular Semiregular Regular

Parameterization-based remeshing

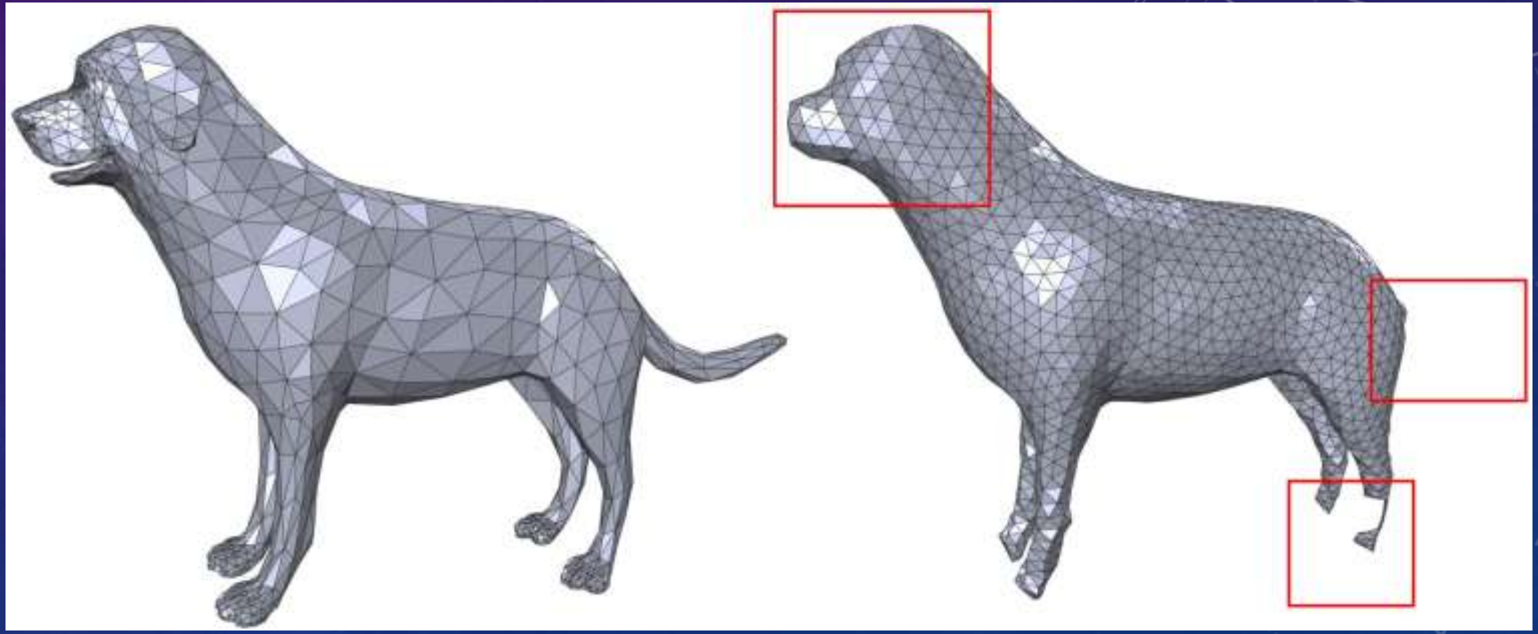
- Low distortion
  - Keeping shapes from the parameter domains
- Cuts
  - Parameterization-based method requires cut paths
  - Visit at least twice



Isotropic triangular meshing

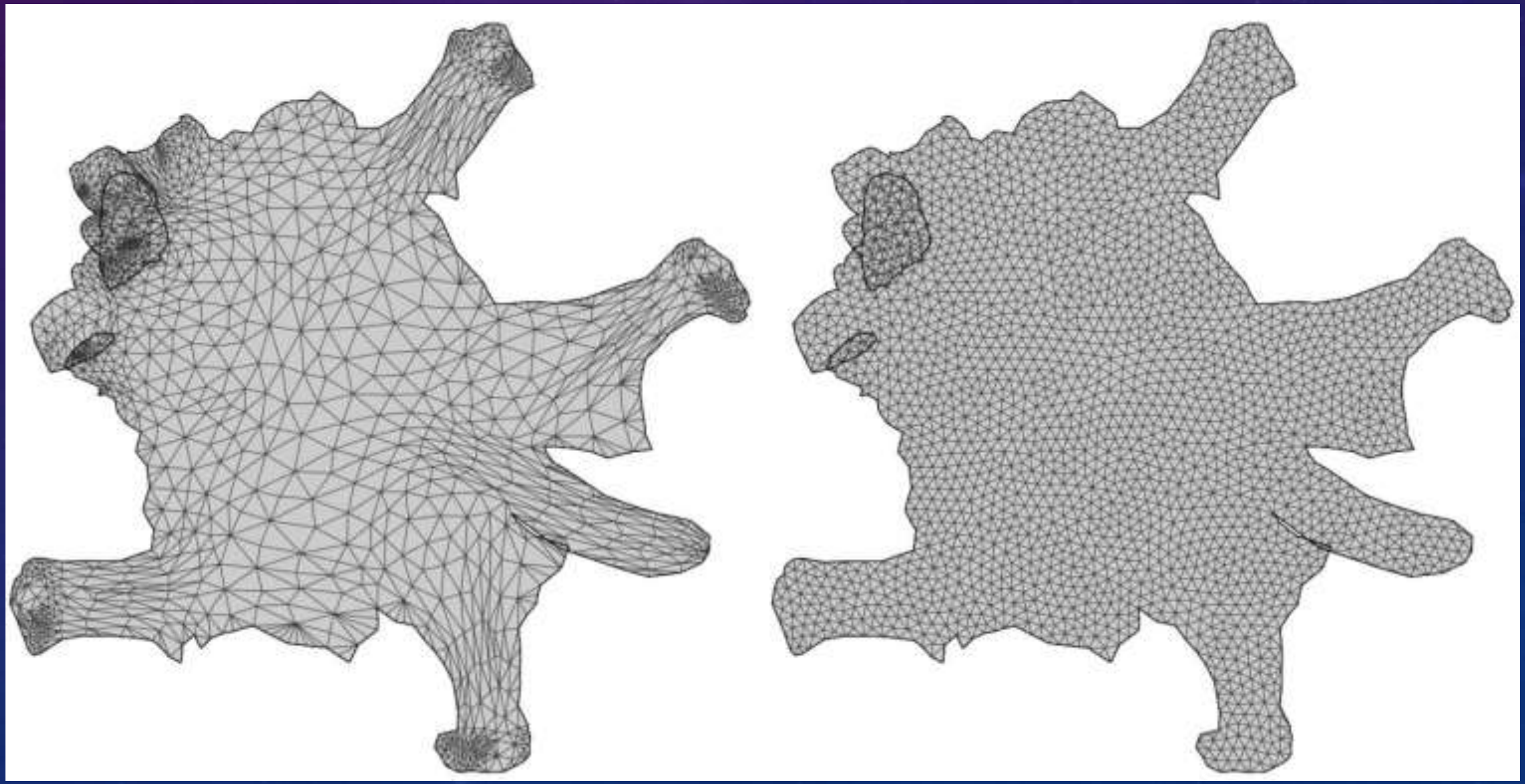
Projection onto the input:

- Time-consuming
- May be incorrect for small-scale features



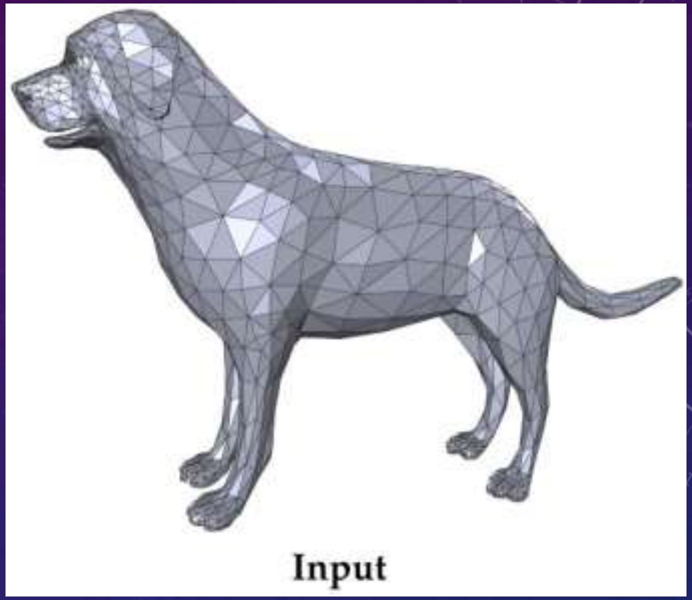
By nearly isometric parameterization

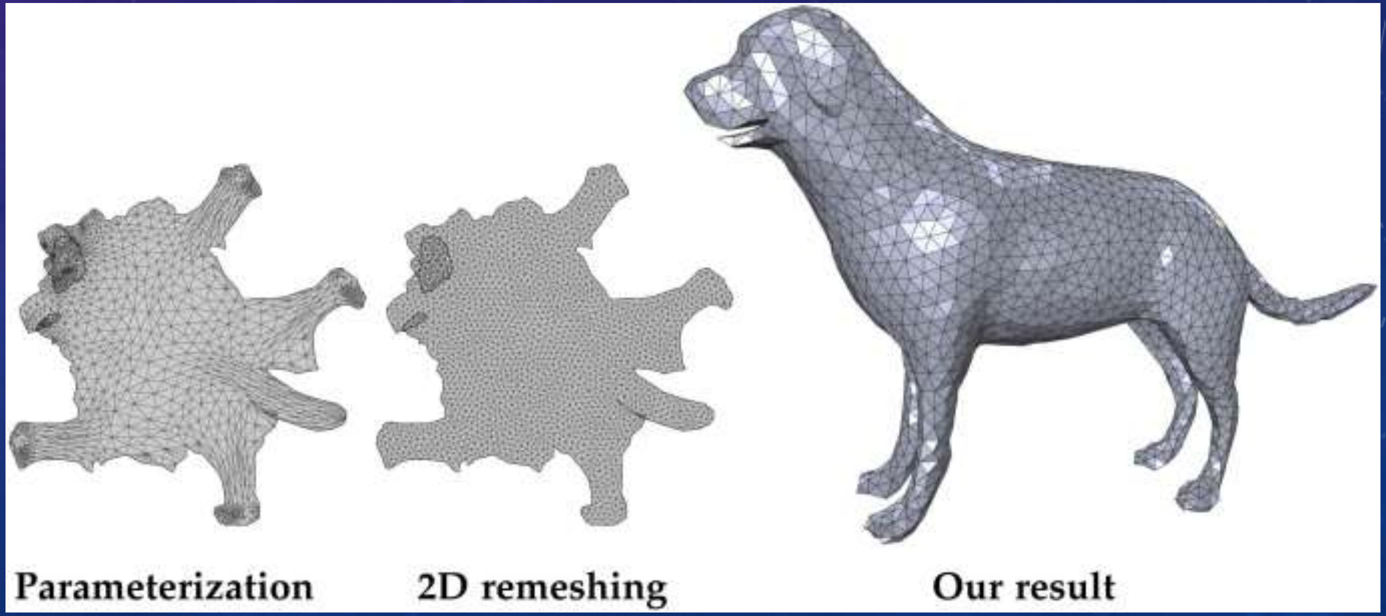
- Remeshing on the plane, no projection



Isotropic remeshing

- Cut the input surface to be disk topology
- Compute parameterizations
- Remesh parameterized domain
- Interpolation on the input





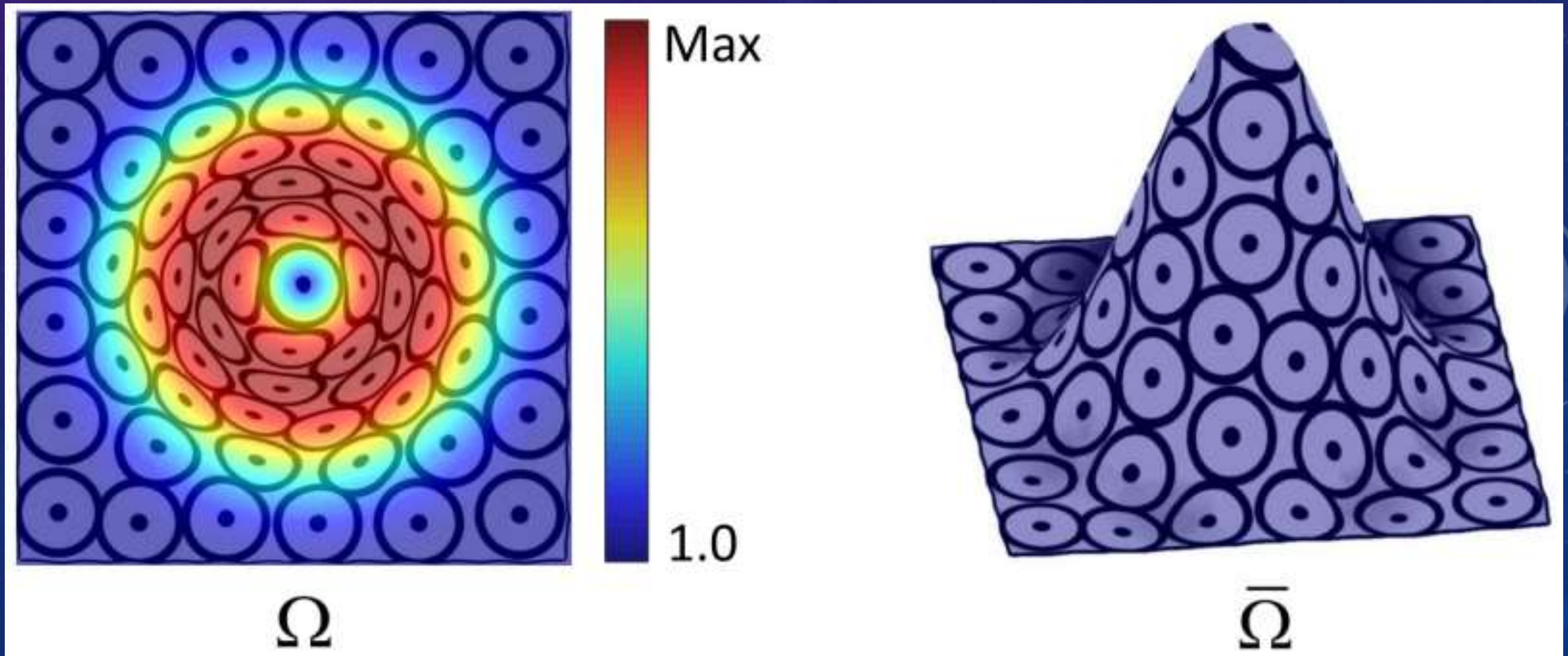
Anisotropic remeshing

- Eigen-decomposition $M(x) = U(x)\Lambda(x)U^T(x)$
- Transformation $\phi = \Lambda^{1/2}(x)U^T(x)$
- Anisotropic remeshing - all edge lengths with metric are as equal as possible

transforms simplex to isotropic space

High-dim isometric embedding

- For an arbitrary metric field $M(x)$ defined on the surface or volume $\Omega \subset \mathbb{R}^m$, there exists a high-d space $\mathbb{R}^n$ $(m < n)$ in which Ω can be embedded with Euclidean metric as $\overline{\Omega} \subset \mathbb{R}^n$.



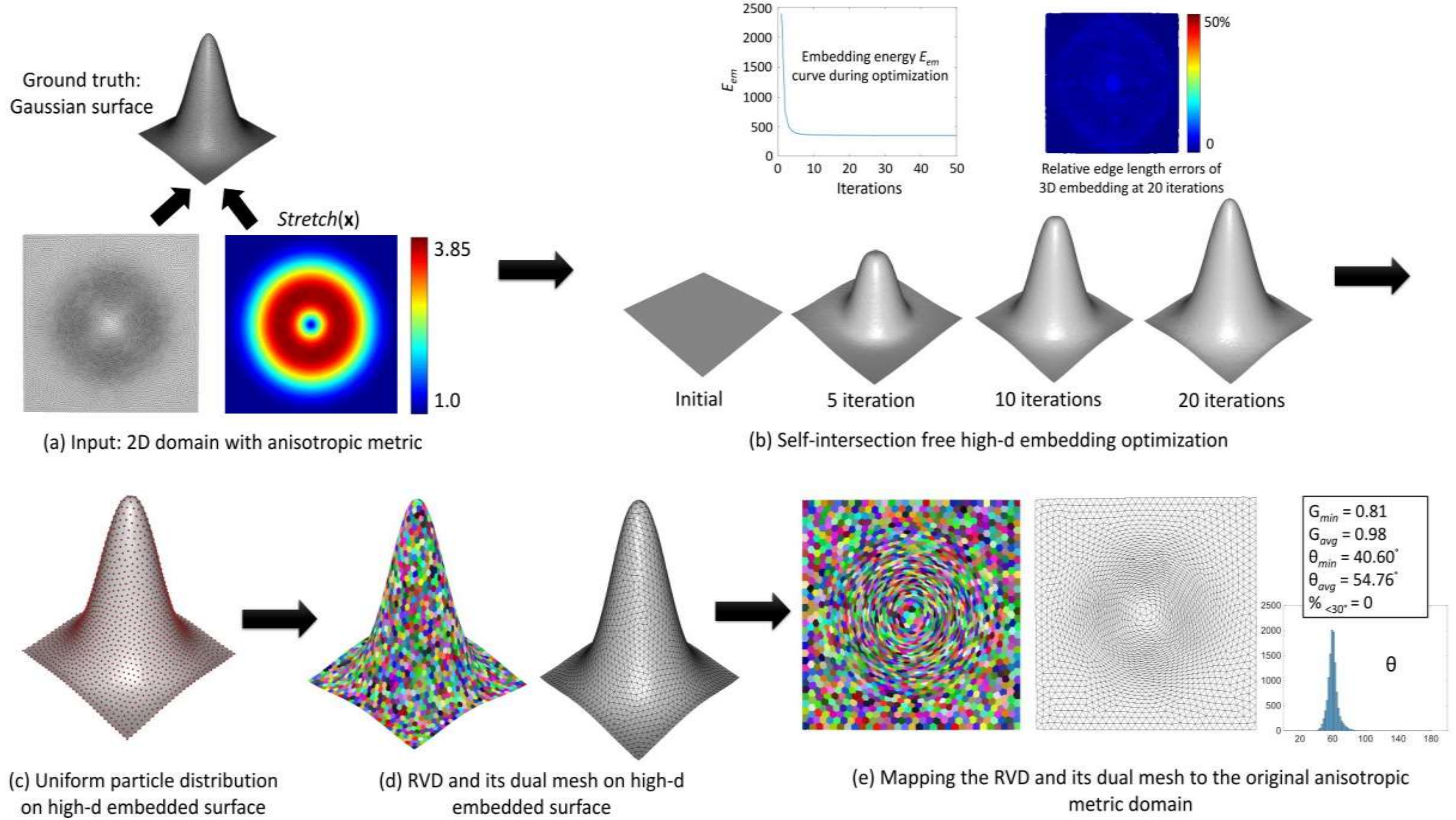
Computing high-dim embedding

local-global solver: $E_{embedding} + \mu E_{smoothing}$

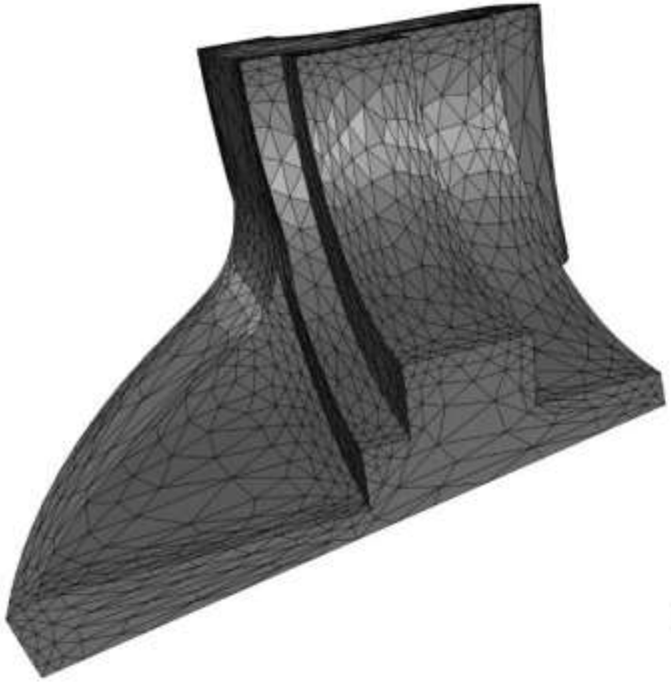
$E_{embedding}$: measure the rigidity, like ARAP

$E_{smoothing}$: measure the smoothness of the embedding

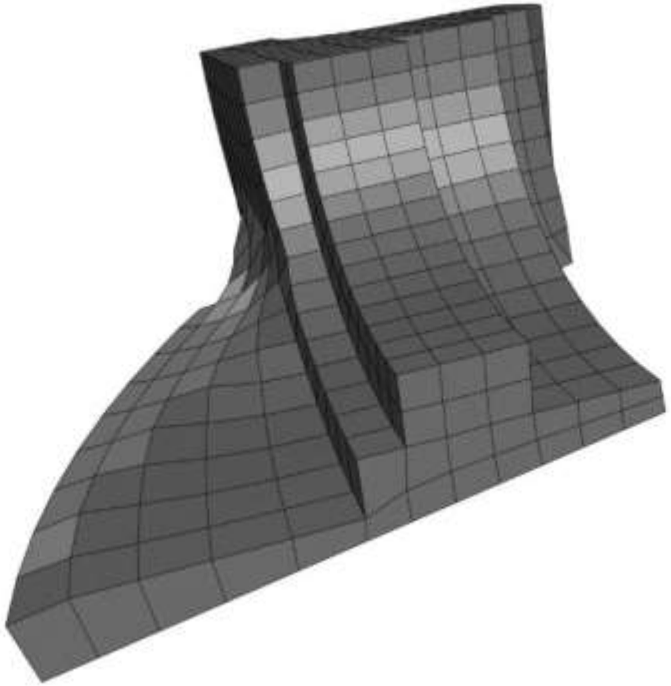
A 3D embedding from a 2D domain with an anisotropic metric


Ground truth:
Gaussian surface
Stretch(x)
3.85
1.0
(a) Input: 2D domain with anisotropic metric
Embedding energy E_{em}
curve during optimization
E_{em}
2500
2000
1500
1000
500
0
0 10 20 30 40 50
Iterations
50%
0
Relative edge length errors of
3D embedding at 20 iterations
Initial
5 iteration
10 iterations
20 iterations
(b) Self-intersection free high-d embedding optimization
(c) Uniform particle distribution
on high-d embedded surface
(d) RVD and its dual mesh on high-d
embedded surface
G_{min} = 0.81
G_{avg} = 0.98
θ_{min} = 40.60°
θ_{avg} = 54.76°
$\%_{<30°}$ = 0
θ
2500
2000
1500
1000
500
0
20 60 100 140 180
(e) Mapping the RVD and its dual mesh to the original anisotropic
metric domain

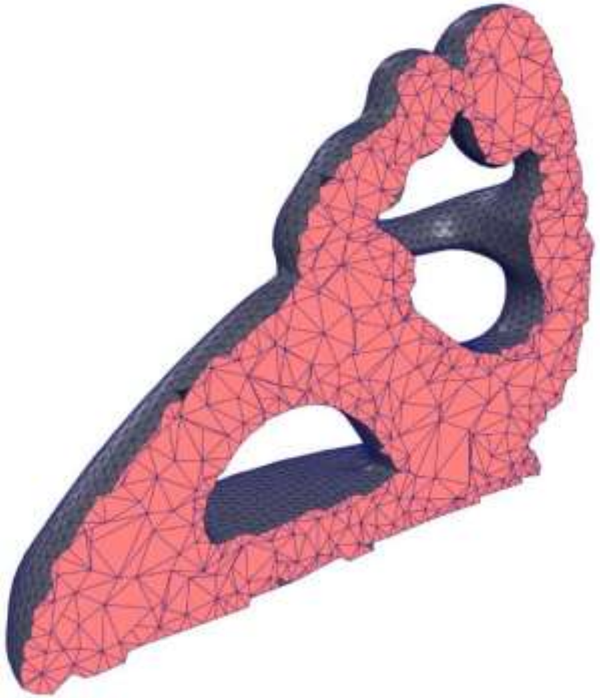
Mesh types



Triangle



Quad



Tet

Hex

Applications

- Atlas generation
- Peeling art
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- Inter-surface mappings

Inter-surface mapping

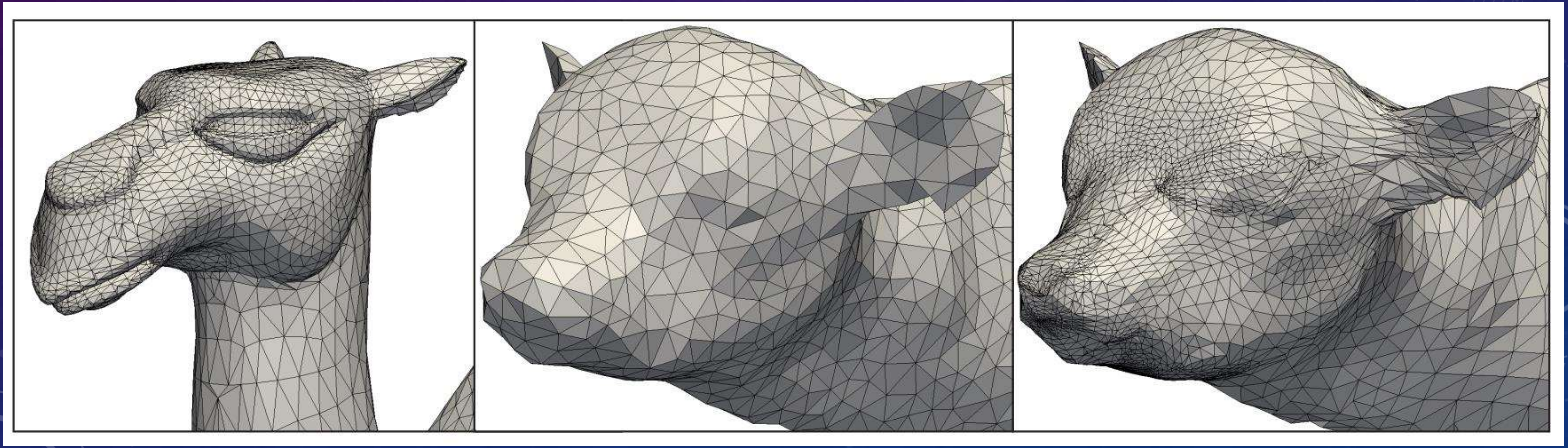
- Cross parameterization
- A one-to-one mapping f between two surfaces M_s and M_t

M_s

M_t

Compatible meshes

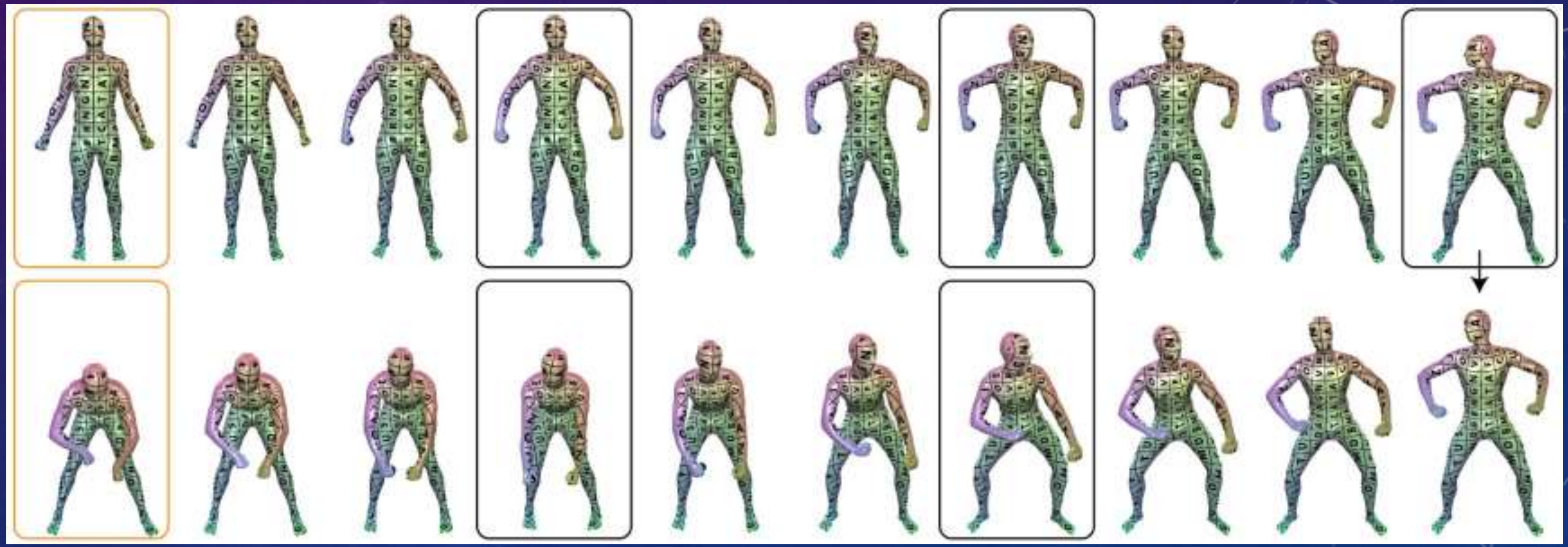
- Meshes with identical connectivity (M_s and $\widehat{M}_t$)



M_s $\widehat{M}_t$ $M_t \cong \widehat{M}_t = f(M_s)$

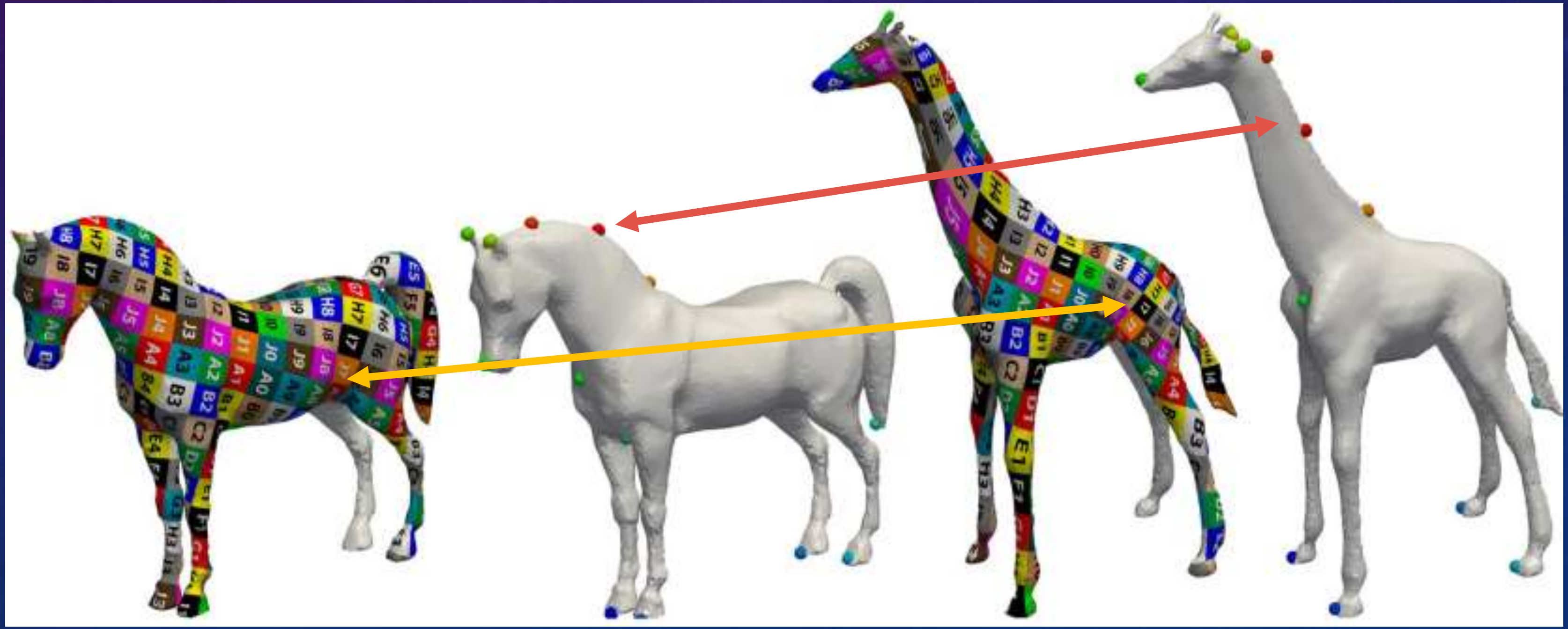
Applications

- Morphing
- Attribute transfer
- ...



Methods

- Input: Two (n) models and some corresponding landmarks
- Output: Bijection and low distortion

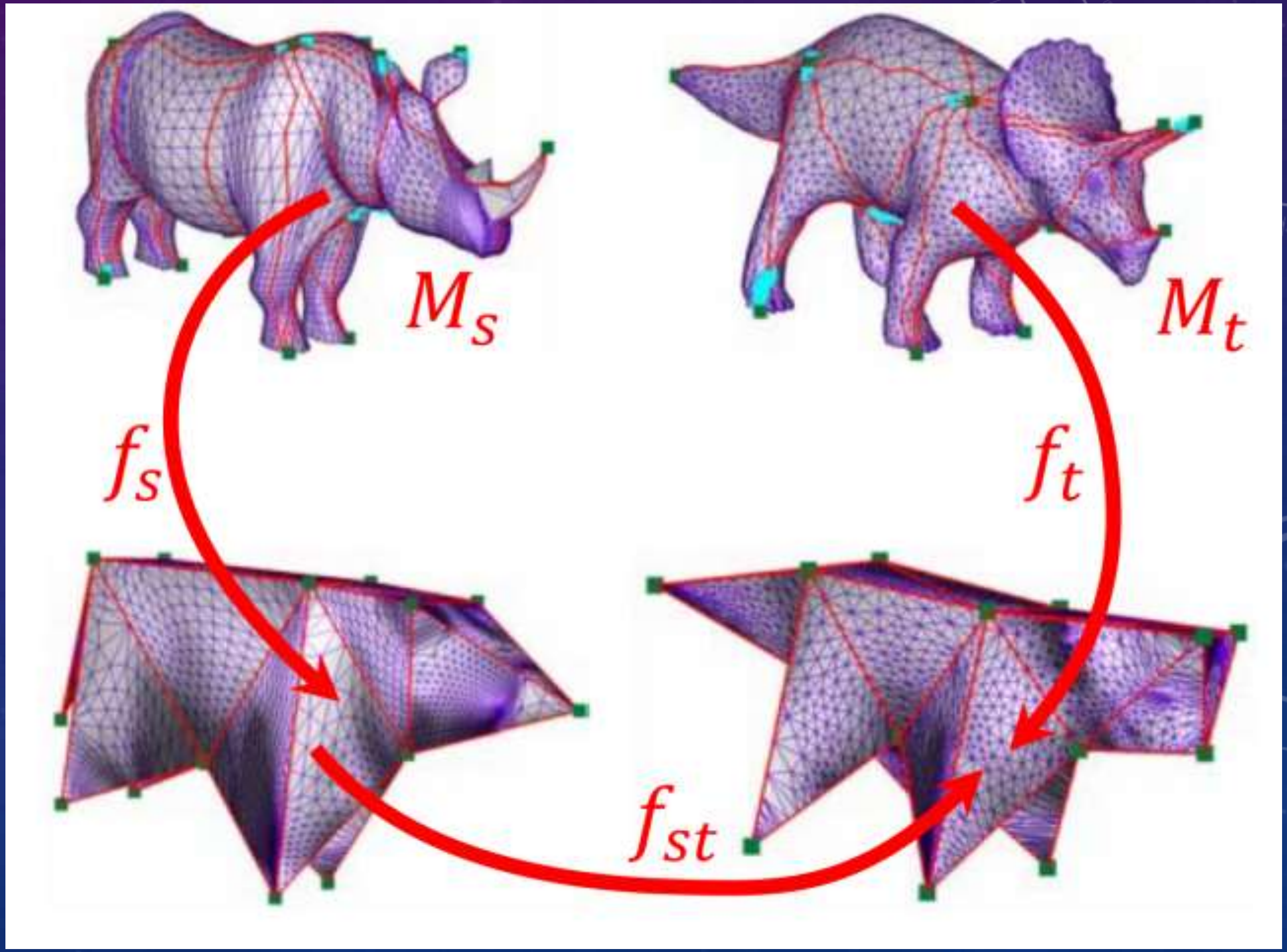


Methods

- Construct a common base domain
  - Topologically identical triangular layouts of the two meshes.
- Compute a low distortion cross-parameterization
  - Each patch is mapped to the corresponding base mesh triangle.
- Compatibly remesh the input models using the parameterizations

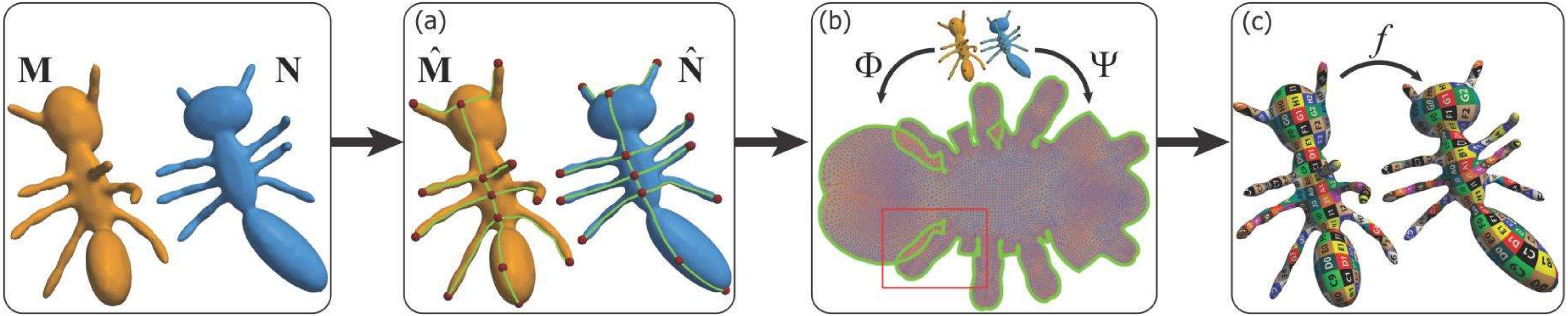
One common base domain

- $f = f_t^{-1} \circ f_{st} \circ f_s$



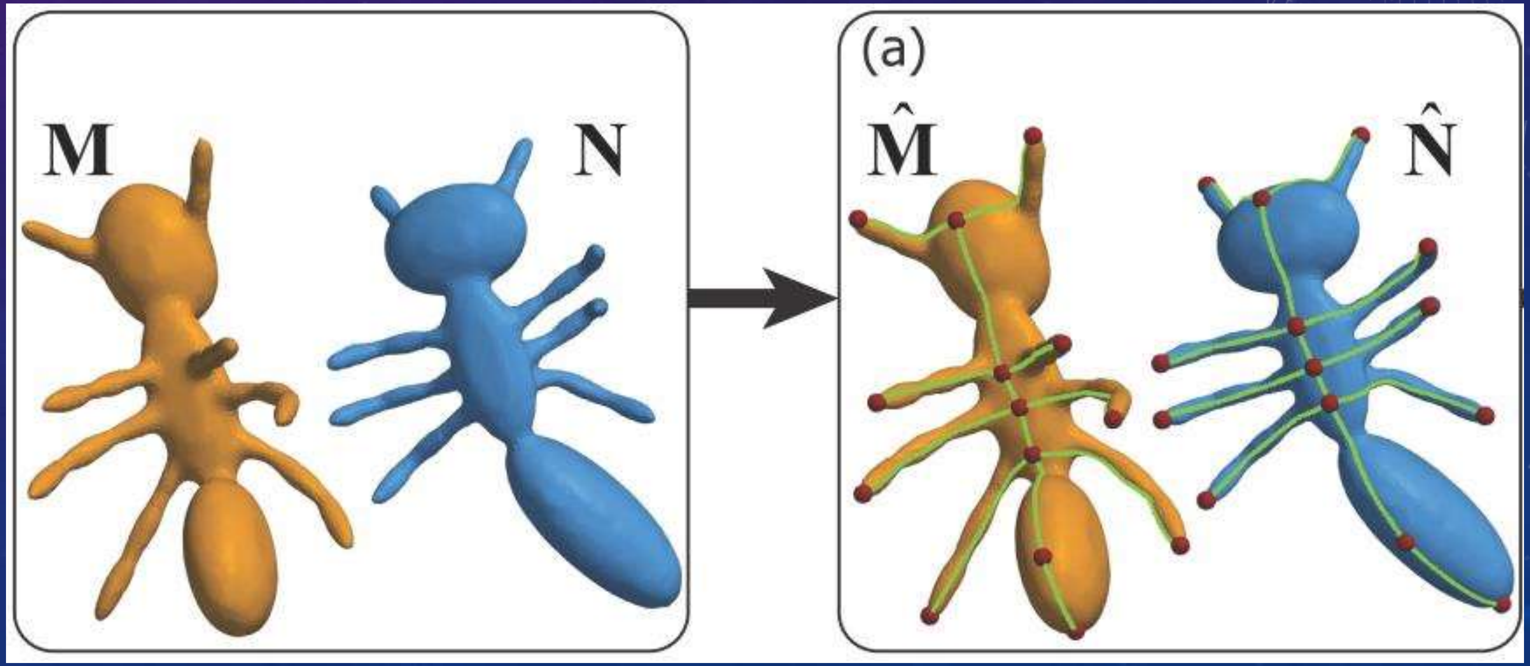
Parameterization domain

- Cutting to disk topology.
- Computing the joint flattenings ϕ, ψ.
- Bijection Lifting



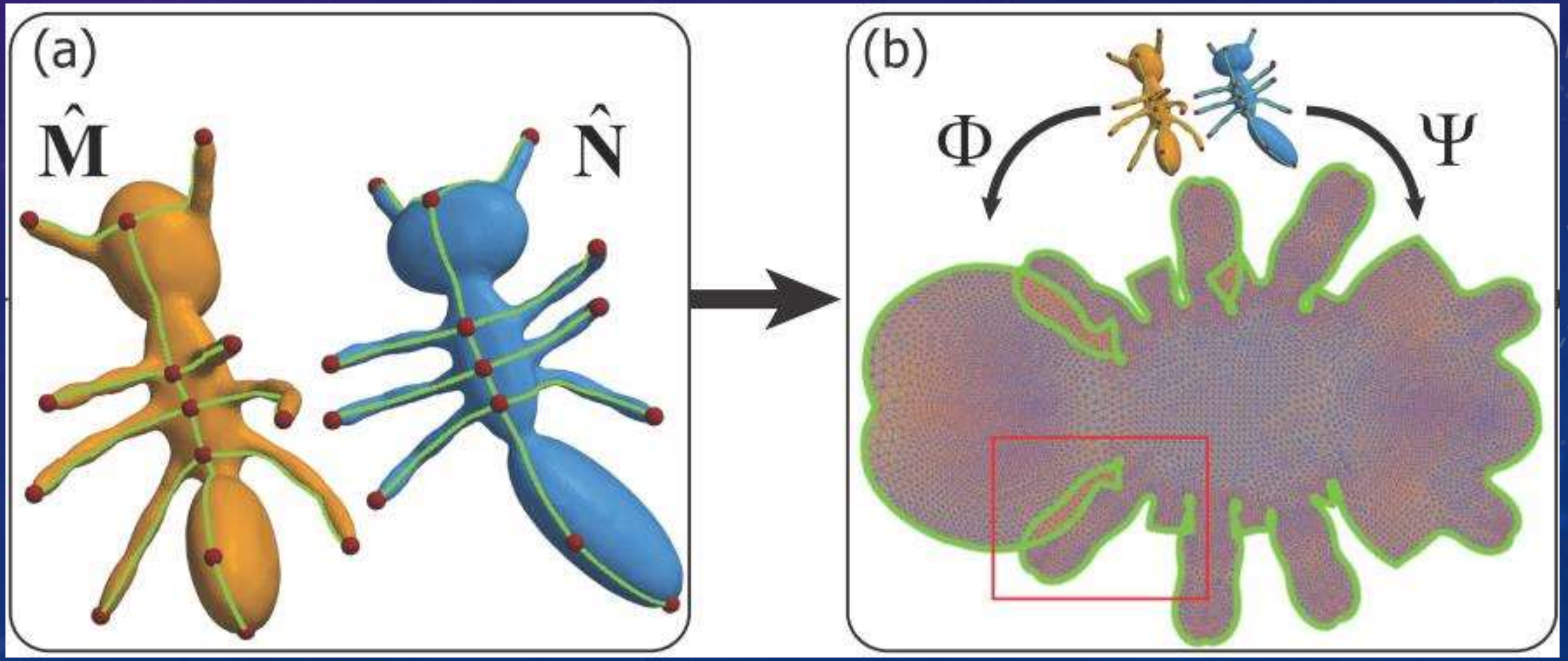
Cutting paths

- Bijective correspondence
  - Shortest path
  - Minimal spanning tree



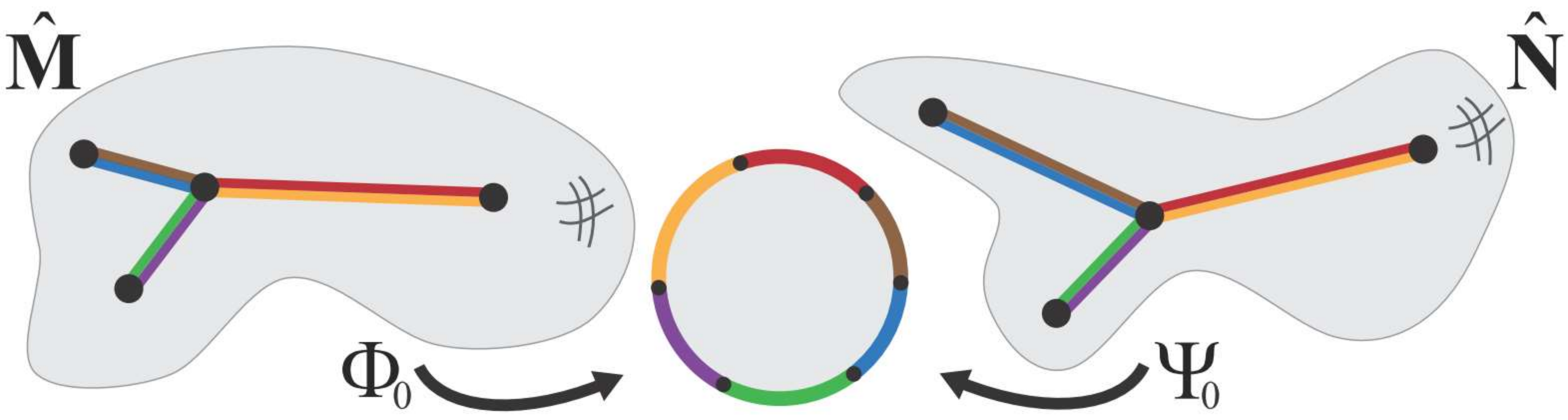
Computing ϕ, ψ

- Constraint
  - Common boundary condition
  - Locally injective



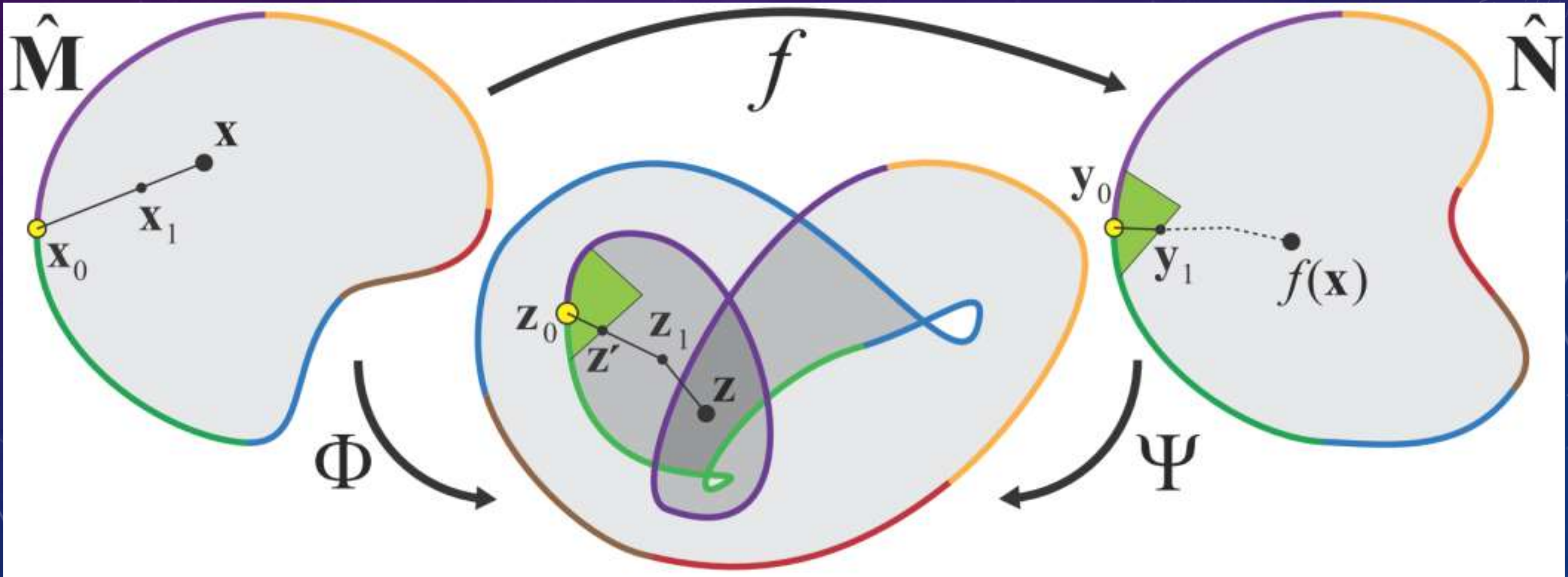
Bijection Lifting

- Bijective parameterizations

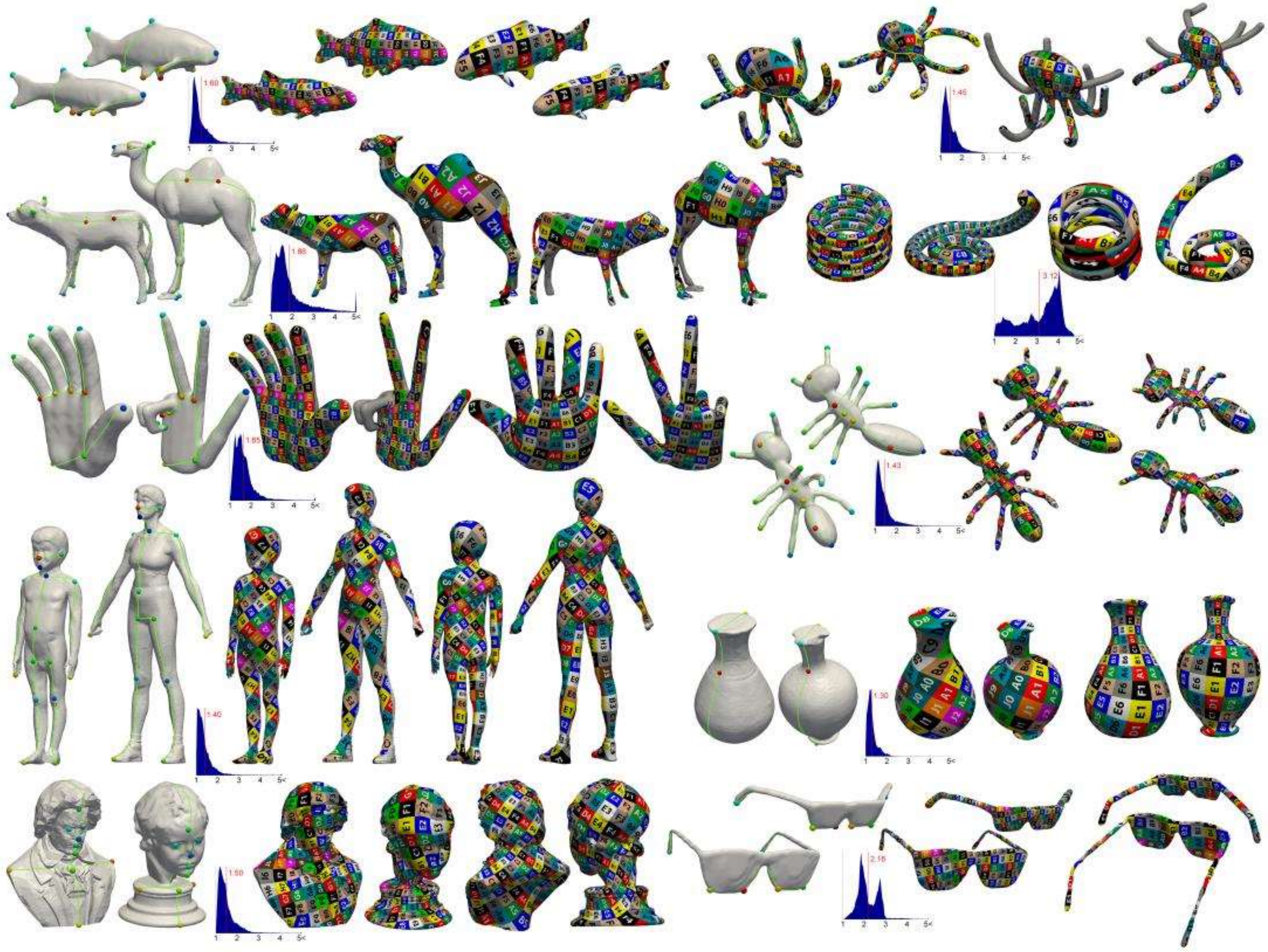


Bijection Lifting

- Only locally injective constrains



Results



Disadvantages

- Cut-dependent