Assignment #6: Spectral Mesh Simplification

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https://qingfang1208.github.io/

Spectral simplification

Algorithm 1: Edge-collapse progressive simplification

Input: mesh $\mathcal{M} = (\mathcal{V}, \mathcal{F})$, target size N, metric $m : \mathcal{V} \times \mathcal{V} \mapsto \mathbb{R}$

Output: simplified mesh $\widetilde{\mathcal{M}} = (\widetilde{\mathcal{V}}, \widetilde{\mathcal{F}})$

$\widetilde{\mathcal{V}} \leftarrow \mathcal{V}$; $\widetilde{\mathcal{F}} \leftarrow \mathcal{F}$; `queue` $\leftarrow \{\}$;

for *edge* $e \in \mathcal{M}$ **do**

  add $(e, m(e))$ to `queue` ;

while $|\widetilde{\mathcal{V}}| > N$ *and* `queue` *not empty* **do**

  $(e, c) \leftarrow$ pop edge e with lowest cost c from `queue` ;

  collapse e (this changes $\widetilde{\mathcal{V}}$ and $\widetilde{\mathcal{F}}$) ;

  for $n \in e$*'s neighbors* **do**

    update n in `queue` ;

Quadric error metric

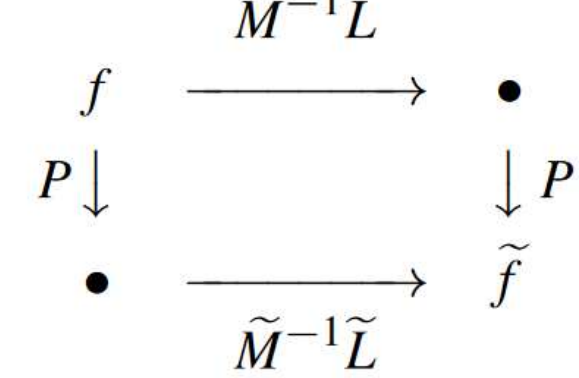
Spectral metric

Spectral metric

- $L, M \in \mathbb{R}^{|V|\times|V|}$ Laplacian and diagonal mass matrix.
- $\tilde{L}, \tilde{M} \in \mathbb{R}^{|\tilde{V}|\times|\tilde{V}|}$ coarse mesh. $F \in \mathbb{R}^{|V|\times K}$ first K eigenvalues.
- $P \in \mathbb{R}^{|\tilde{V}|\times|V|}$ the fine-to-coarse restriction matrix.

$$E = \left\| PM^{-1}LF - \tilde{M}^{-1}\tilde{L}PF \right\|_{\tilde{M}}^2, \quad \|X\|_{\tilde{M}}^2 = tr\left(X^T\tilde{M}X\right)$$

$$= \mathrm{diag}\left(\tilde{M}\right)^T \mathrm{diag}(XX^T) = \sum_v \tilde{M}_v \|\mathrm{row}_v(X)\|^2$$



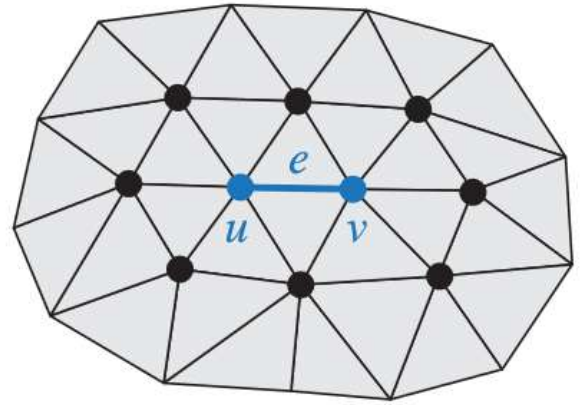


Figure 4: *When collapsing e (blue), only the 1-ring entries of E_w (black) change.*

Spectral metric

$$E = \left\| PM^{-1}LF - \tilde{M}^{-1}\tilde{L}PF \right\|_{\tilde{M}}^2 = \sum_v \tilde{M}_v \left\| \mathrm{row}_v\left(PM^{-1}LF - \tilde{M}^{-1}\tilde{L}PF\right) \right\|^2$$

$$\mathcal{H} = \{u, v\} \cup \mathcal{N}_1(u, v)$$

$$\begin{aligned}
\mathrm{cost}(e) &= E^{after} - E^{before} \\
&= \sum_{w \in \mathcal{H}} E_w^{after} + \sum_{w \notin \mathcal{H}} E_w^{after} - \sum_{w \in \mathcal{H}} E_w^{before} - \sum_{w \notin \mathcal{H}} E_w^{before} \\
&= \sum_{w \in \mathcal{H}} E_w^{after} - \sum_{w \in \mathcal{H}} E_w^{before}
\end{aligned}$$

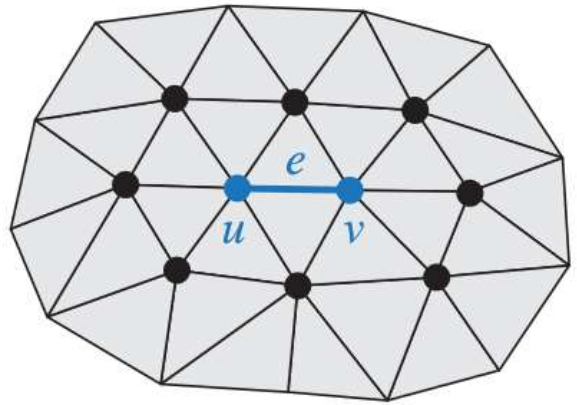
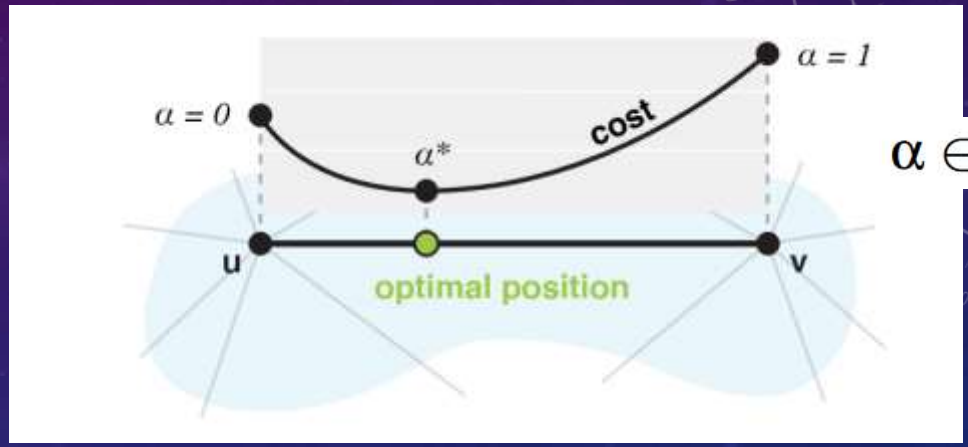


Figure 4: *When collapsing e (blue), only the 1-ring entries of E_w (black) change.*

Merged vertex optimization

- Merge at the edge center
- 1D quadratic approximation
- 3D quadratic approximation



$$\alpha \in \{0, 0.5, 1\}$$

Strategy	$\|\cdot\|_L$	$\|\cdot\|_D$	Time
(i) middle	1.0	1.0	**1.0**
(ii) on edge	**0.7**	**0.6**	2.1
(iii) unrestricted	0.8	0.9	4.9

$$\text{Laplacian commutativity: } \|C\|_L^2 = \frac{\|C\Lambda - \tilde{\Lambda}C\|^2}{\|C\|^2}$$

$$\text{Orthonormality: } \|C\|_D^2 = \|C^\top C - \mathrm{Id}\|^2$$

Evaluation

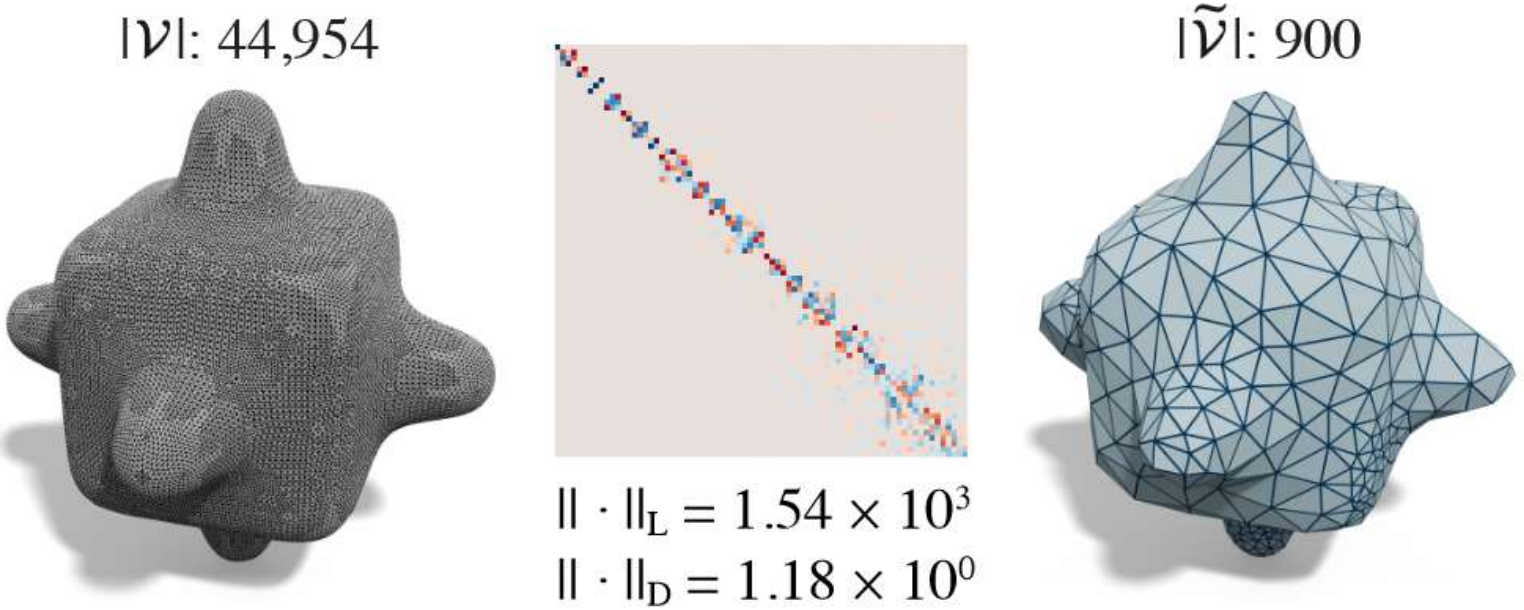
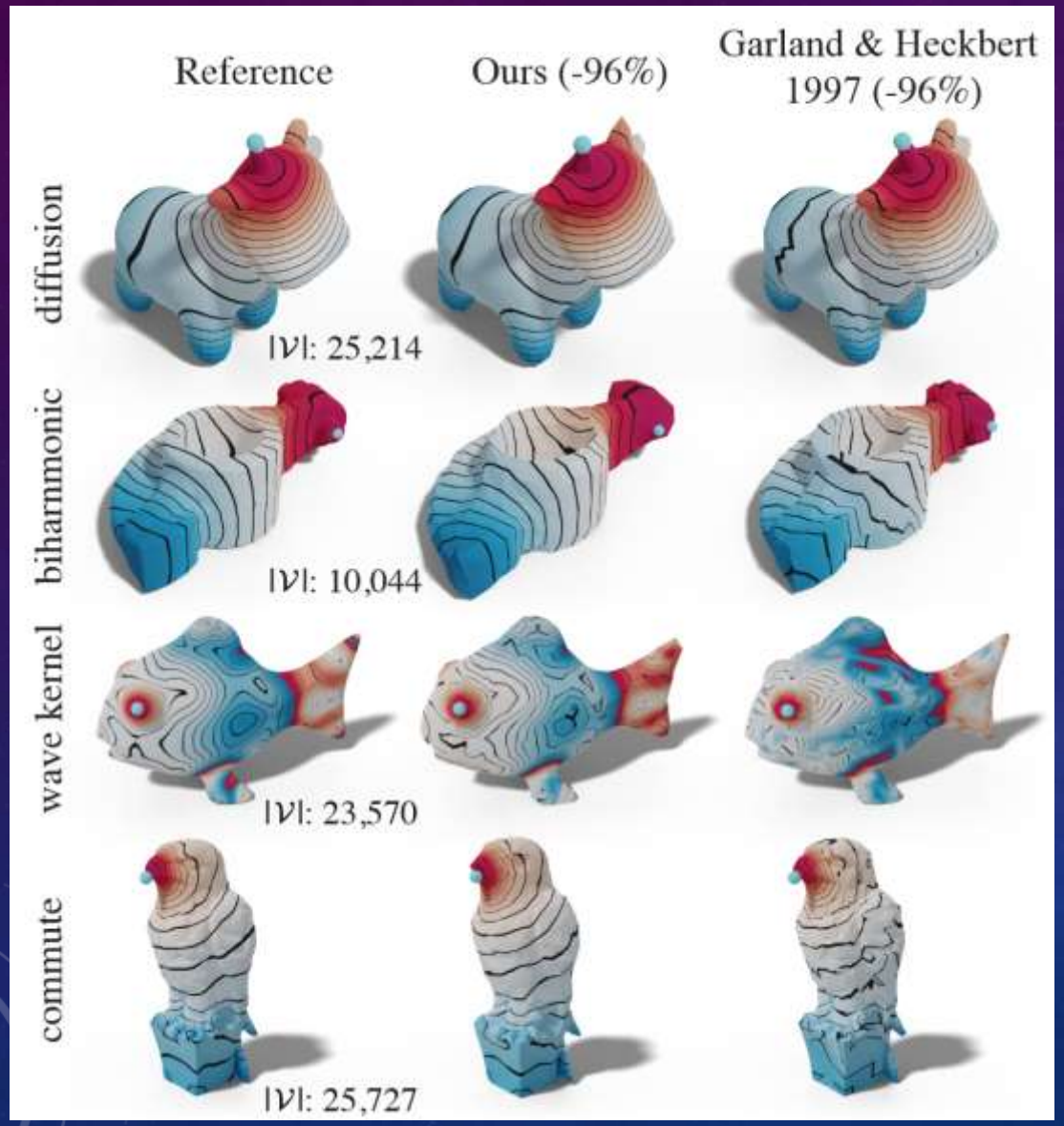
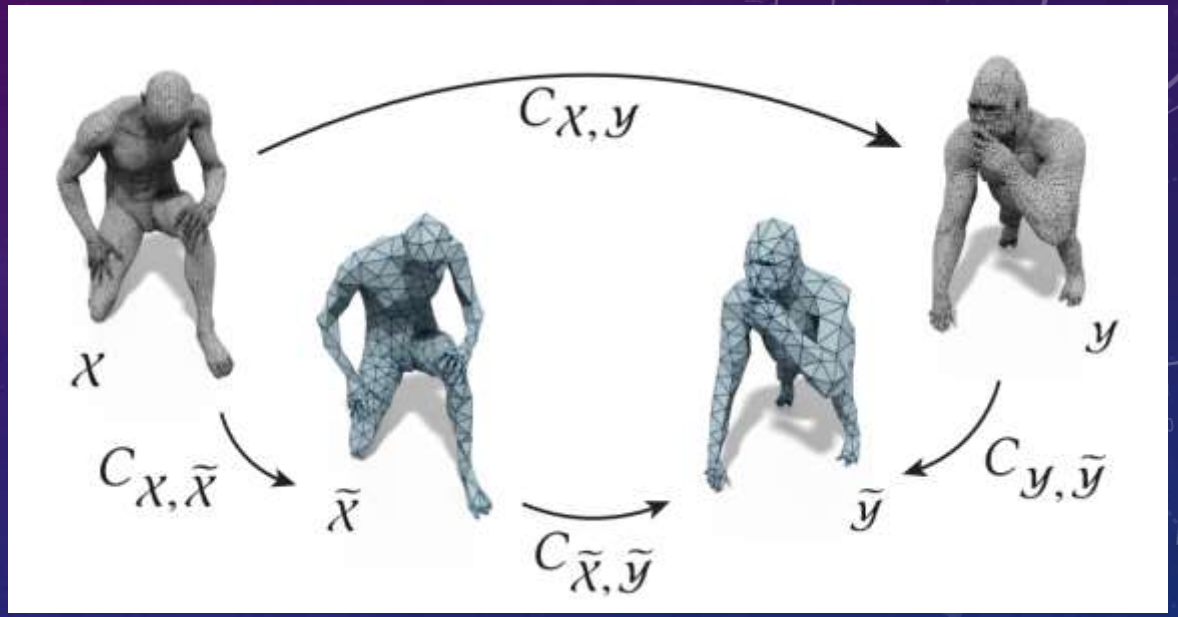
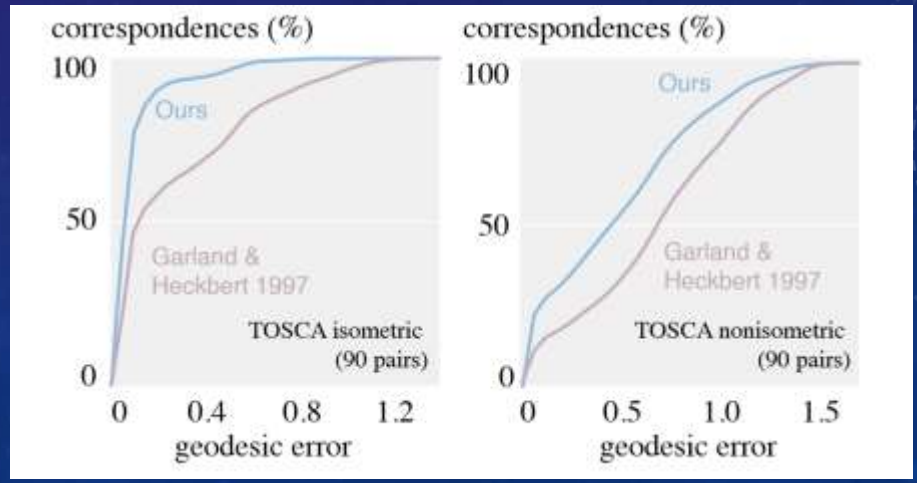


Figure 6: *The functional map from the reduction should be block-diagonal following the multiplicity of the eigenvalues.*


Reference
Ours (-96%)
Garland & Heckbert 1997 (-96%)
diffusion
|V|: 25,214
biharmonic
|V|: 10,044
wave kernel
|V|: 23,570
commute
|V|: 25,727


$C_{X,Y}$
X
Y
$C_{X,\tilde{X}}$
$\tilde{X}$
$C_{\tilde{X},\tilde{Y}}$
$\tilde{Y}$
$C_{Y,\tilde{Y}}$

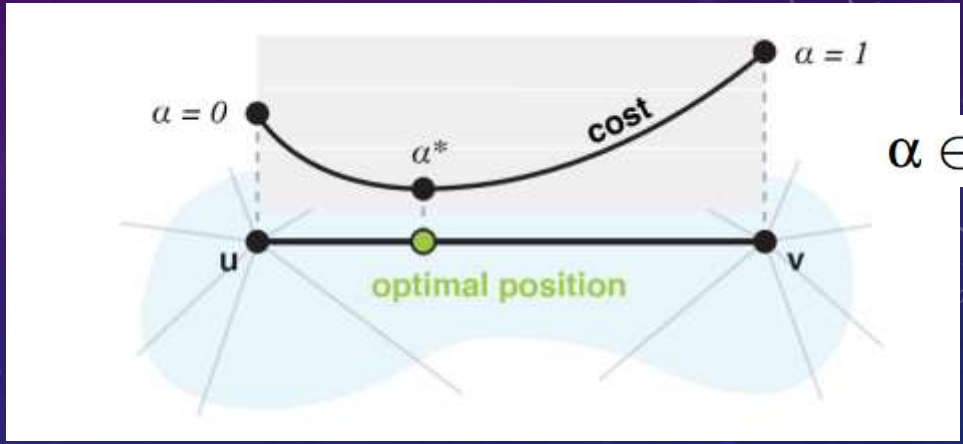

correspondences (%)
100
50
0
Ours
Garland & Heckbert 1997
TOSCA isometric (90 pairs)
0
0.4
0.8
1.2
geodesic error
correspondences (%)
100
50
0
Ours
Garland & Heckbert 1997
TOSCA nonisometric (90 pairs)
0
0.5
1.0
1.5
geodesic error

$N = 30{,}000$

$N = 600$

Assignment requirements

- Alg : Spectral mesh simplification
  - 1D quadratic approximation
- Email: ID_name_homework#6.zip
- Deadline: 2024.05.08, 23:59



$\alpha \in \{0, 0.5, 1\}$