

The background features a dark blue gradient with faint, light blue technical diagrams. On the left side, there is a large circular scale with numerical markings from 140 to 260 in increments of 10. Several circular paths with arrows are scattered across the slide, suggesting a technical or engineering context.

# Assignment #1: Simplification with Quadric Error Metric

USTC, 2024 Spring

Qing Fang, [fq1208@mail.ustc.edu.cn](mailto:fq1208@mail.ustc.edu.cn)

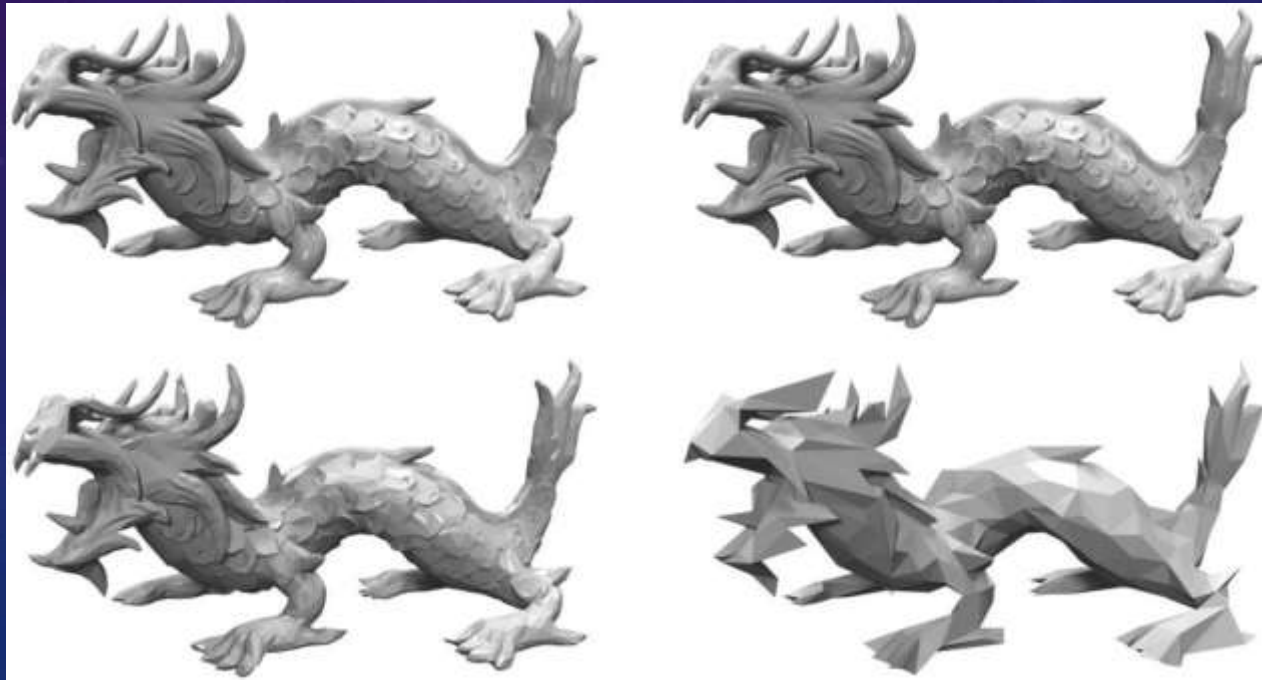
<https://qingfang1208.github.io/>

Background

The background is a gradient of dark blue to purple, overlaid with a pattern of small white stars. Several technical diagrams are visible: a circular gauge with a scale from 0 to 210 and an arrow pointing to approximately 180; a circular diagram with concentric rings and arrows indicating a clockwise cycle; and a circular diagram with concentric rings and arrows indicating a counter-clockwise cycle.

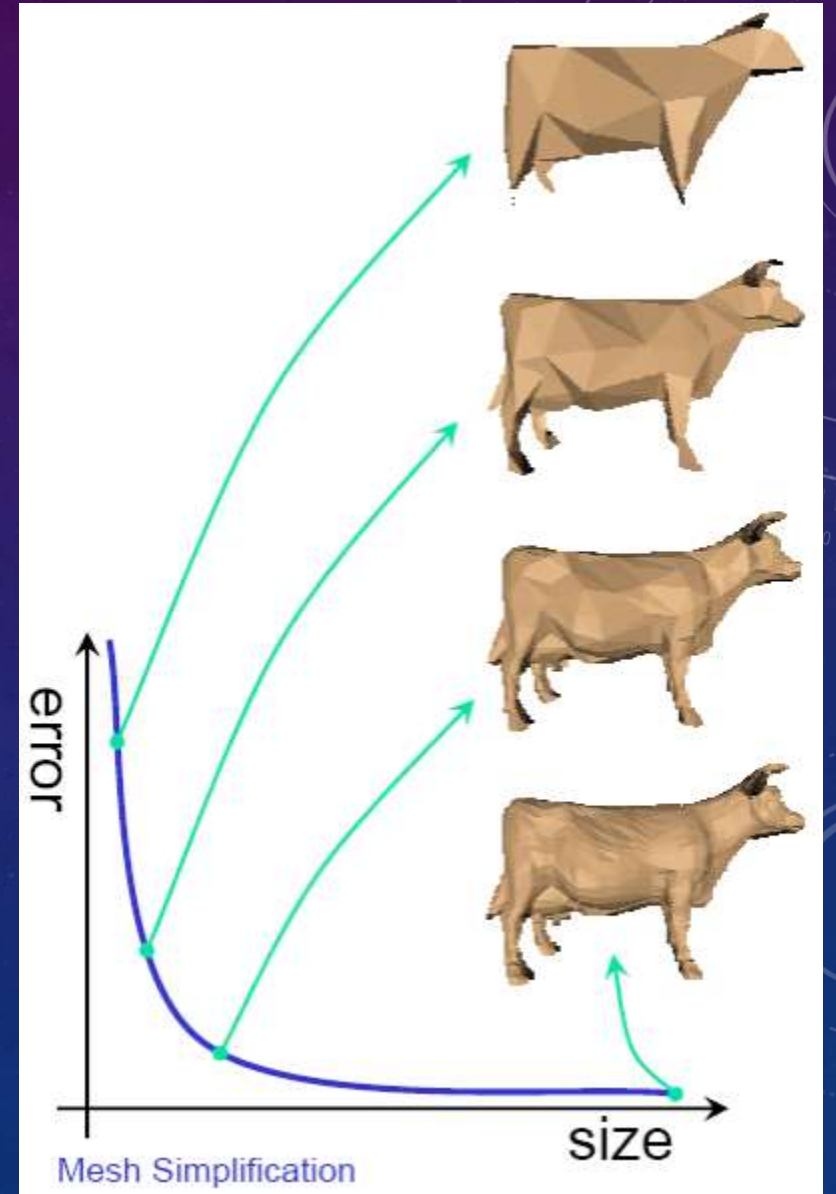
# Definition of simplification

- Transform a given polygonal mesh into another mesh with fewer faces, edges, and vertices.



# User-defined quality criteria

- Geometric error





# User-defined quality criteria

- Geometric error
- Other criteria (**curvature**)

2053

1500

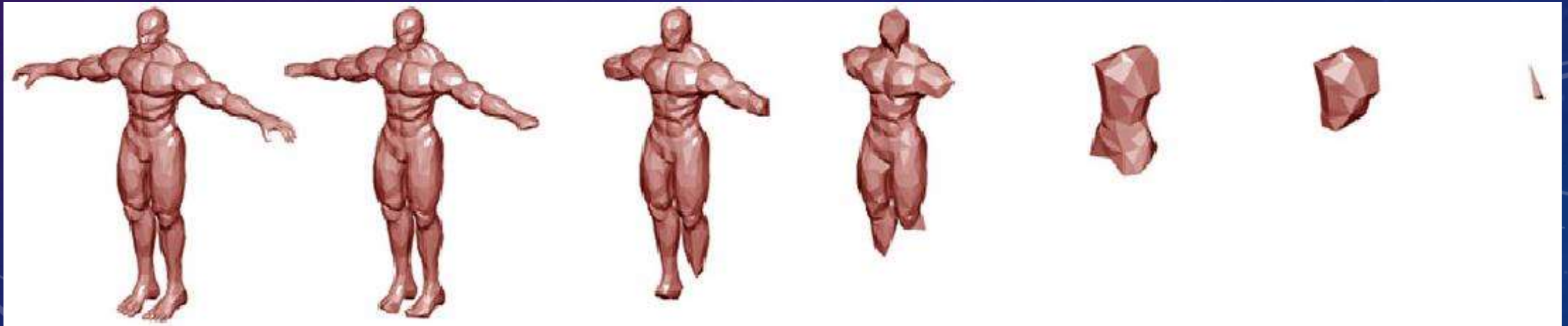
1000

500

100

50

4



Without curvature

# User-defined quality criteria

- Approximation error
- Other criteria (**curvature**)

2053

1500

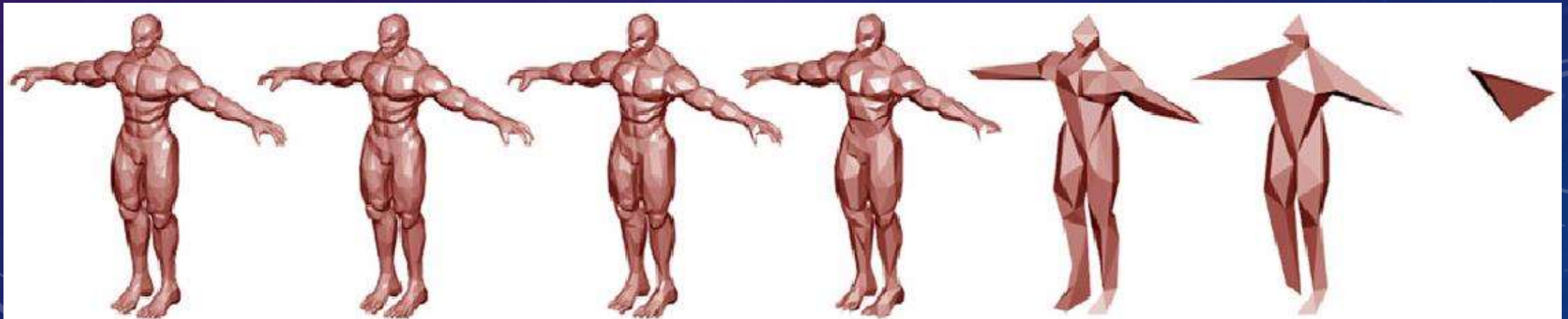
1000

500

100

50

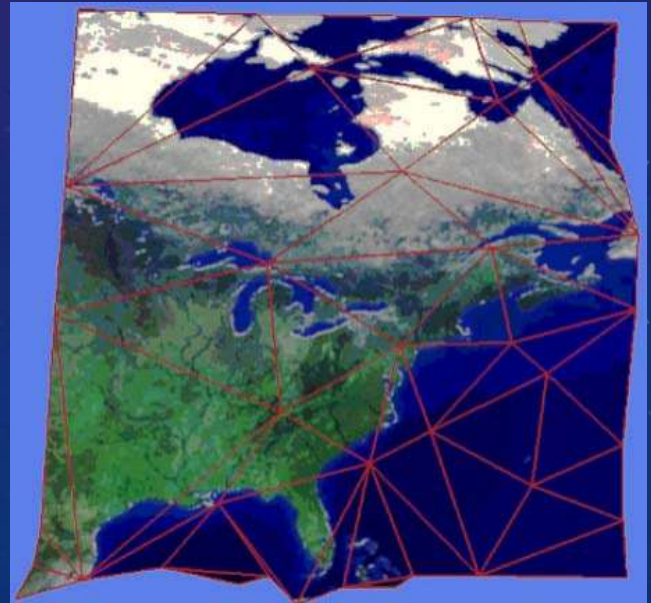
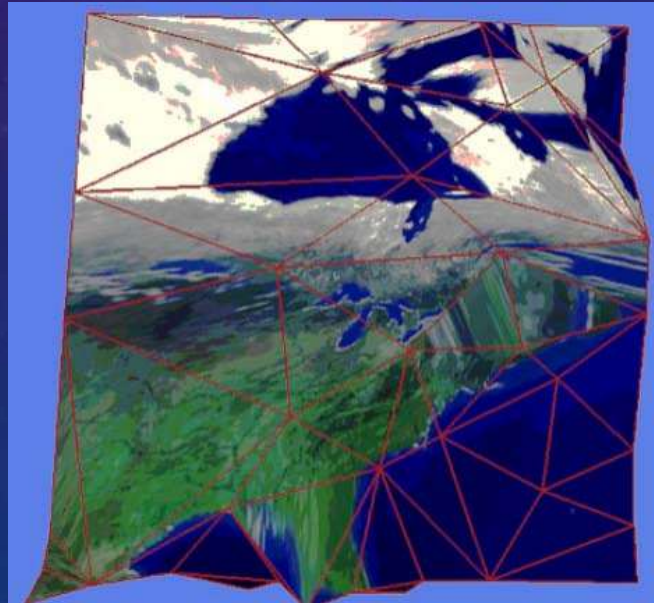
4



With curvature

# User-defined quality criteria

- Approximation error
- Other criteria (curvature, **texture**)





# Local operations

The background features a dark blue gradient with a subtle pattern of white stars. Overlaid on this are several technical diagrams. In the top right, there is a large circular gauge with a scale from 0 to 210 and a needle pointing to approximately 180. Below it is a smaller circular diagram with concentric rings and arrows. In the bottom left, there is another circular diagram with a dashed outer ring and a solid inner ring, with an arrow pointing left. A faint circular diagram is also visible in the top left corner.



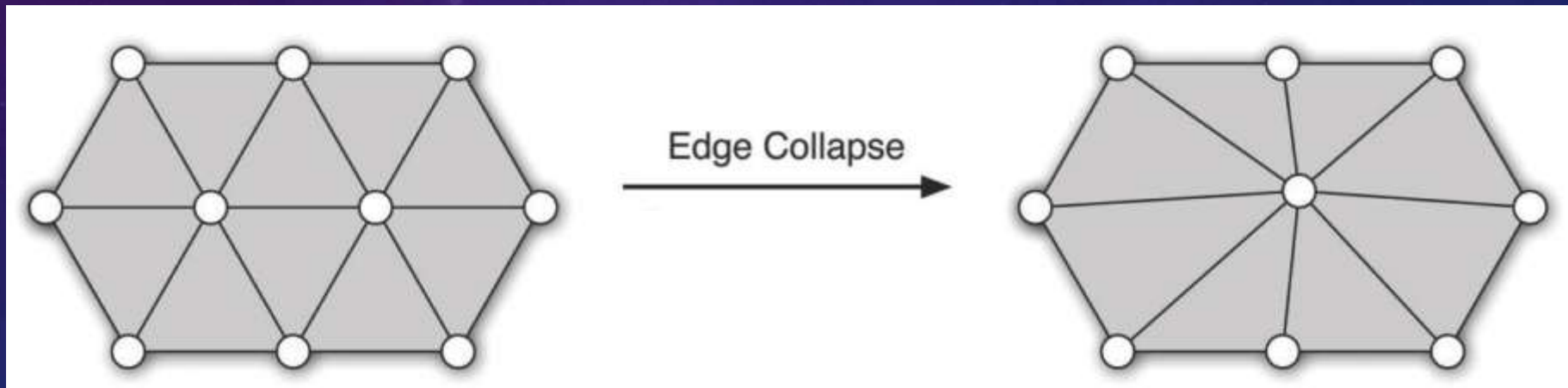
# Topological operations

- Decimation (**vertex removal**)



# Topological operations

- Decimation (**edge collapse**)



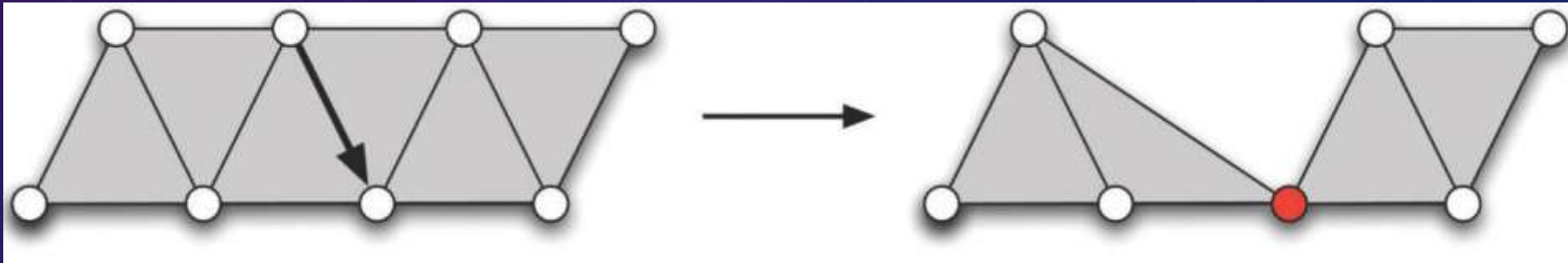
# Topological operations

- Decimation (**half-edge collapse**)



# Illegal (half-)edge collapses

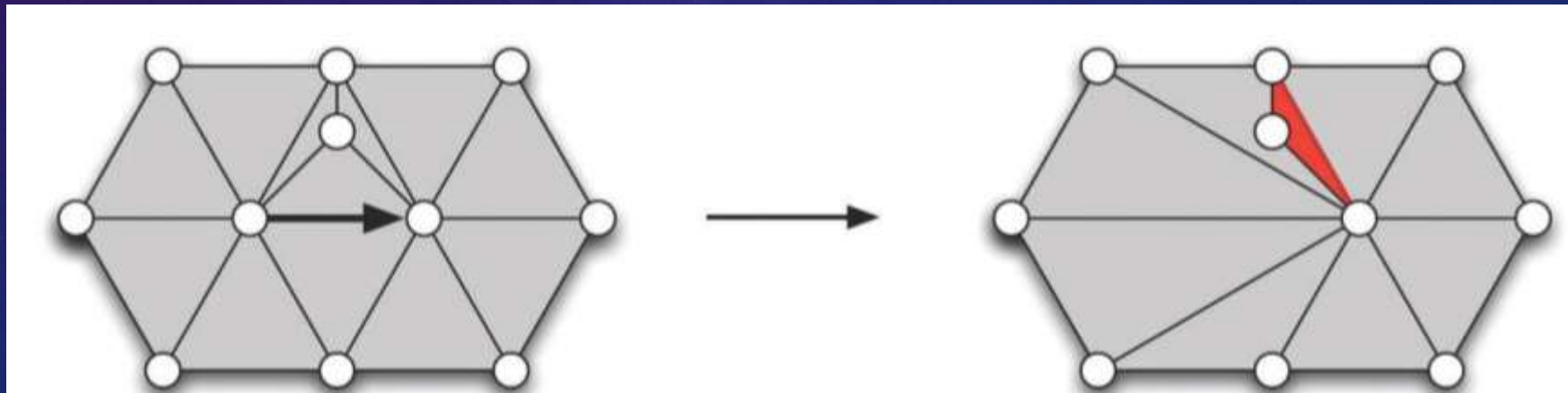
- If  $p$  and  $q$  are boundary vertices, then edge  $(p,q)$  should be a boundary edge.





# Illegal (half-)edge collapses

- If  $i$  and  $j$  are boundary vertices, then edge  $(i, j)$  should be a boundary edge.
- For each  $k$  incident to both  $i$  and  $j$ ,  $\{i, j, k\}$  should be the vertices of a triangle.



# Quadric error metric



# Quadratic error metric (QEM)

- The squared distance of a point  $x$  from the plane  $P$ :

$$d(x, P)^2 = (n^T x - d)^2, d = n^T y$$

Denote  $\bar{x} = (x, 1)$  and  $\bar{n} = (n, -d)$ , then

$$d(x, P)^2 = (\bar{n}^T \bar{x})^2 = \bar{x}^T \bar{n} \bar{n}^T \bar{x} \triangleq \bar{x}^T Q_P \bar{x}$$

$$P = (y, n)$$


# Quadratic error metric (QEM)

- For vertex  $i$ ,  $Q_i \triangleq \sum_{ijk} Q_{ijk}$

$$d(x, Q_i)^2 = \bar{x}^T Q_i \bar{x} = (x^T, 1) Q_i \begin{pmatrix} x \\ 1 \end{pmatrix}$$

When  $x = v_i$ ,  $d(x, Q_i)^2 = 0$



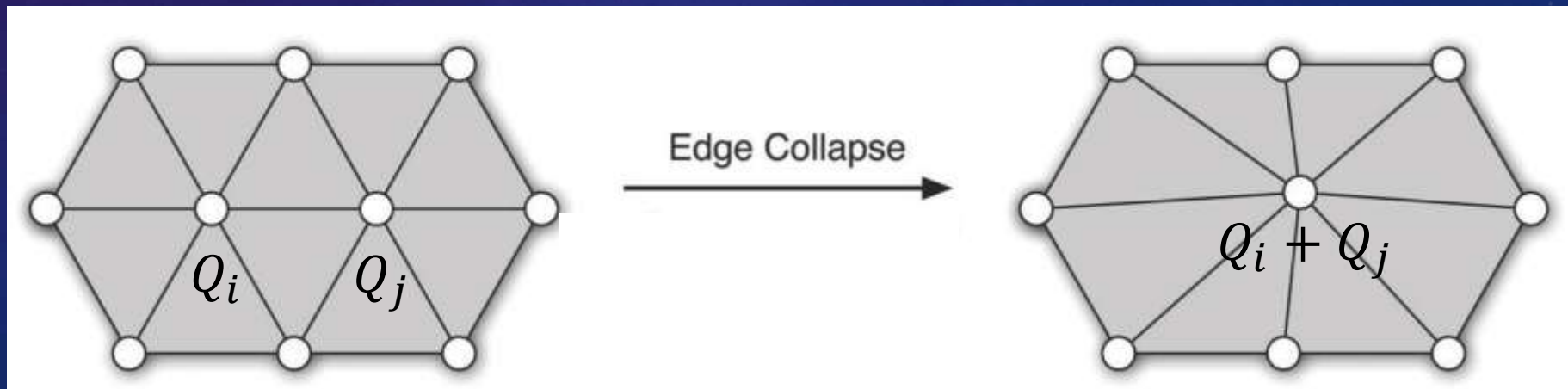


# Quadric error metric (QEM)

- When edge  $(i, j)$  collapses,

$$d^2 = d(x, Q_i)^2 + d(x, Q_j)^2 = \bar{x}^T (Q_i + Q_j) \bar{x}$$

- New position  $x = \arg \min \bar{x}^T (Q_i + Q_j) \bar{x}$



# QEM algorithm

Surface simplification using quadric error metrics  
[Garland & Heckbert 1997]

- Input: a mesh
- Output: a simplified mesh

Initialization:

- Compute the matrices  $Q_i$  for each vertex  $i$
- Compute the optimal contraction target  $v$  for each edge  $(i, j)$

While  $N_V > N$  and  $Cost_{min} < t$

- The error  $v^T(Q_i + Q_j)v$  becomes the cost of the edge  $(i, j)$
- Place all the edges in a priority queue keyed on cost with minimum cost edge at the top.
- Remove the edge of the least cost from the heap, collapse this edge, and update the costs of all edges involving.

End

# Extension

Vertex attributes Become added dimensions

- Color  $(x, y, z, r, g, b)$
- Texture  $(x, y, z, u, v)$
- Normal  $(x, y, z, n_x, n_y, n_z)$
- ...



# Assignment requirements

- QEM algorithm
- Email: ID\_name\_homework#1.zip
  - Pdf : Input + parameter + output
  - Source code (no exe)
- Deadline: 2024.03.10, 23:59