Assignment #1: Simplification with Quadric Error Metric

USTC, 2024 Spring

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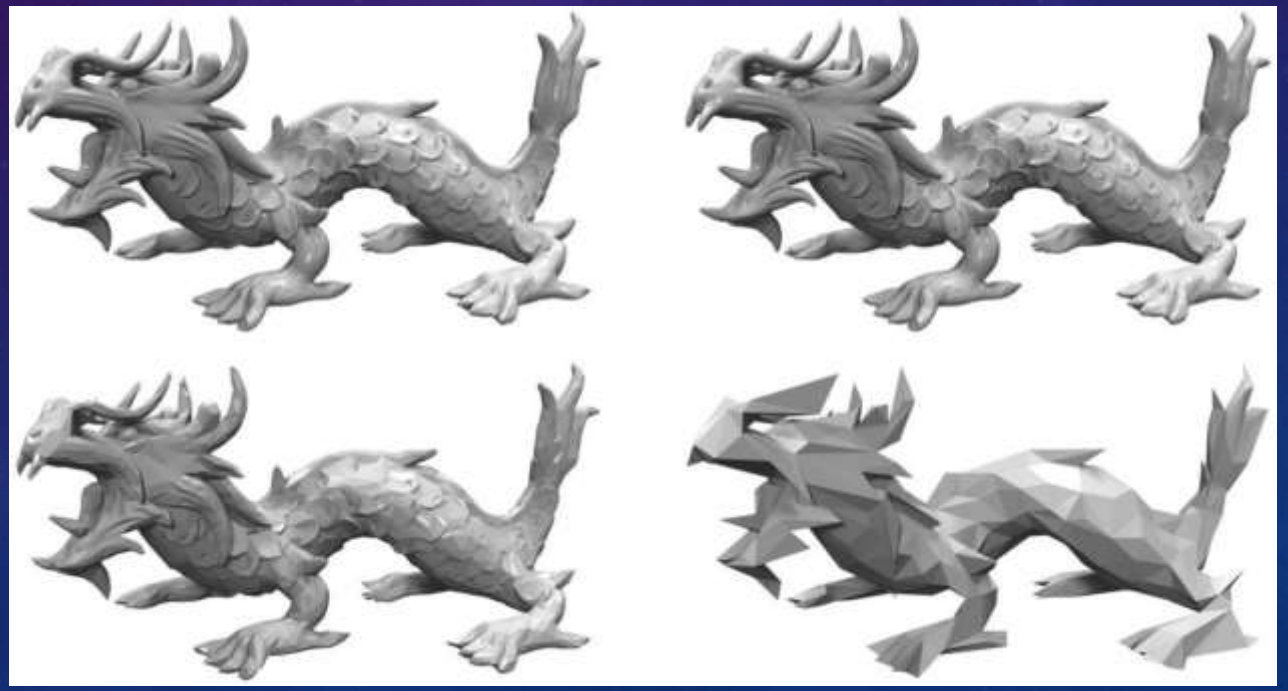
https://qingfang1208.github.io/

Background

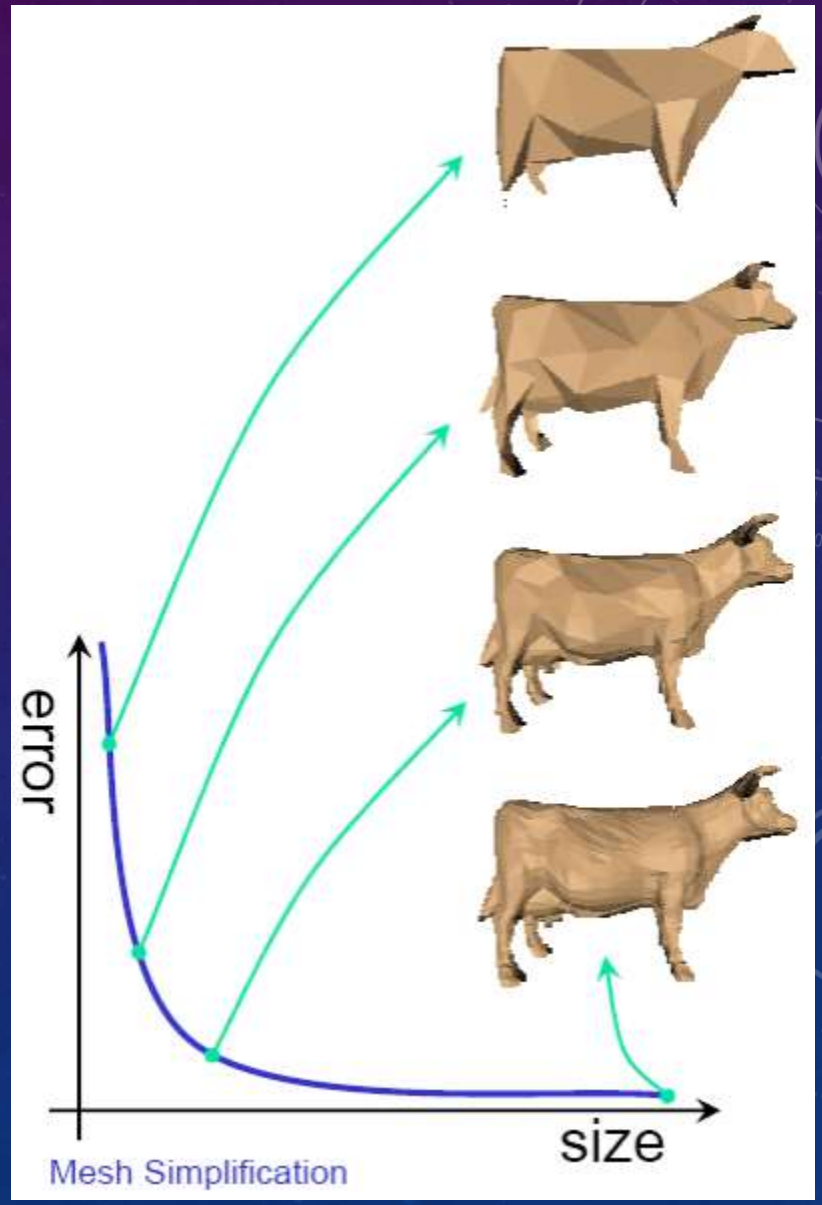
Definition of simplification

- Transform a given polygonal mesh into another mesh with fewer faces, edges, and vertices.



User-defined quality criteria

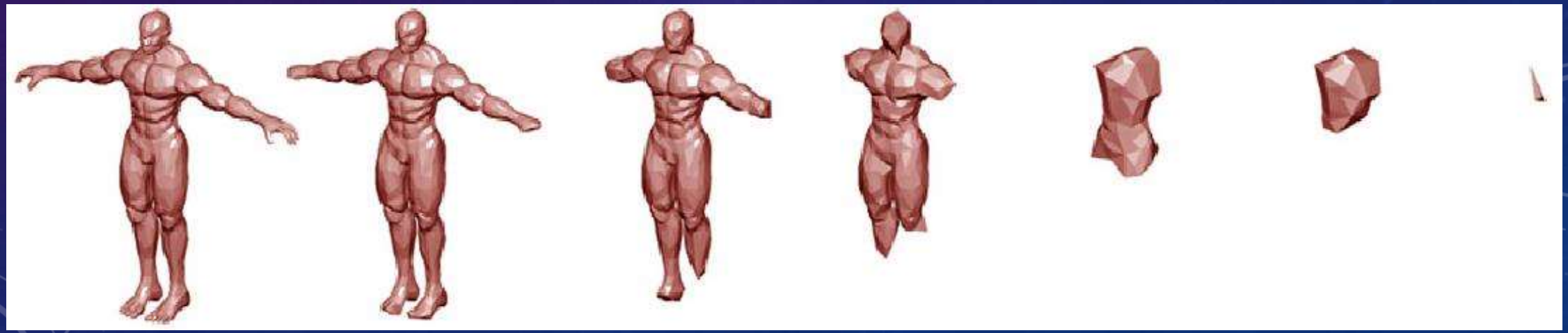
- Geometric error



User-defined quality criteria

- Geometric error
- Other criteria (curvature)

2053 1500 1000 500 100 50 4

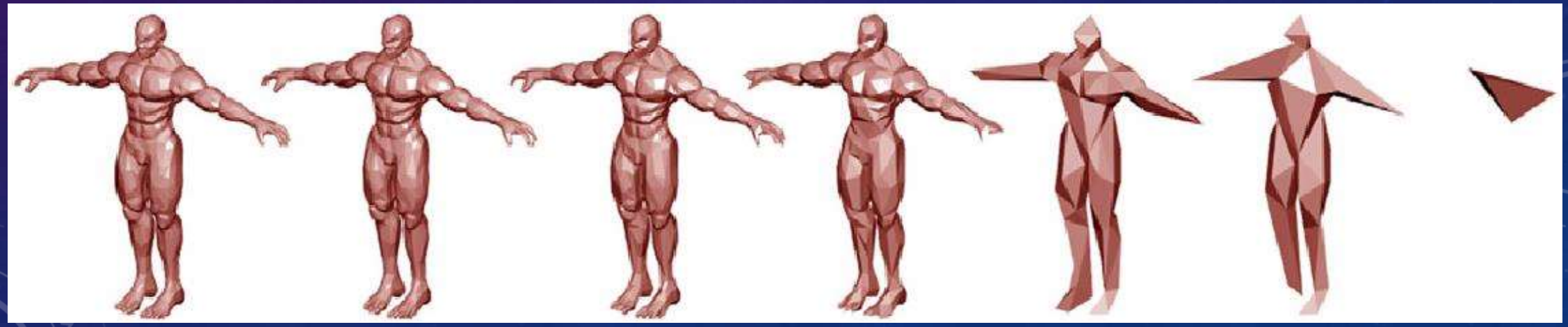


Without curvature

User-defined quality criteria

- Approximation error
- Other criteria (curvature)

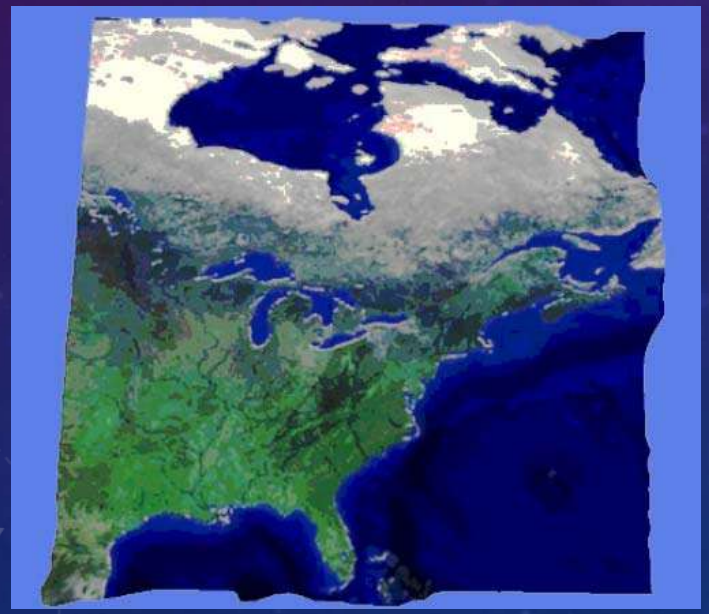
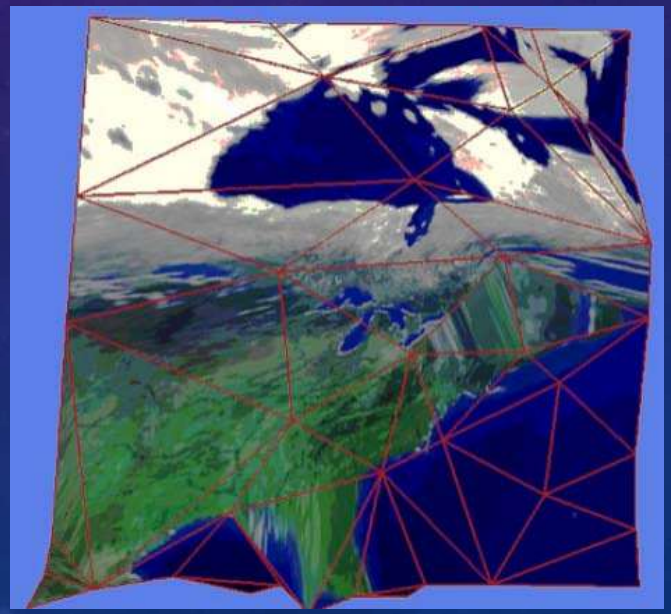
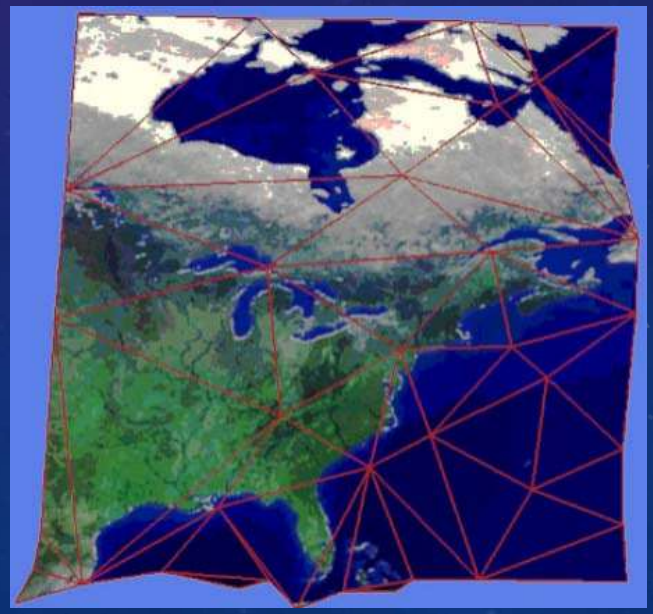
2053 1500 1000 500 100 50 4



With curvature

User-defined quality criteria

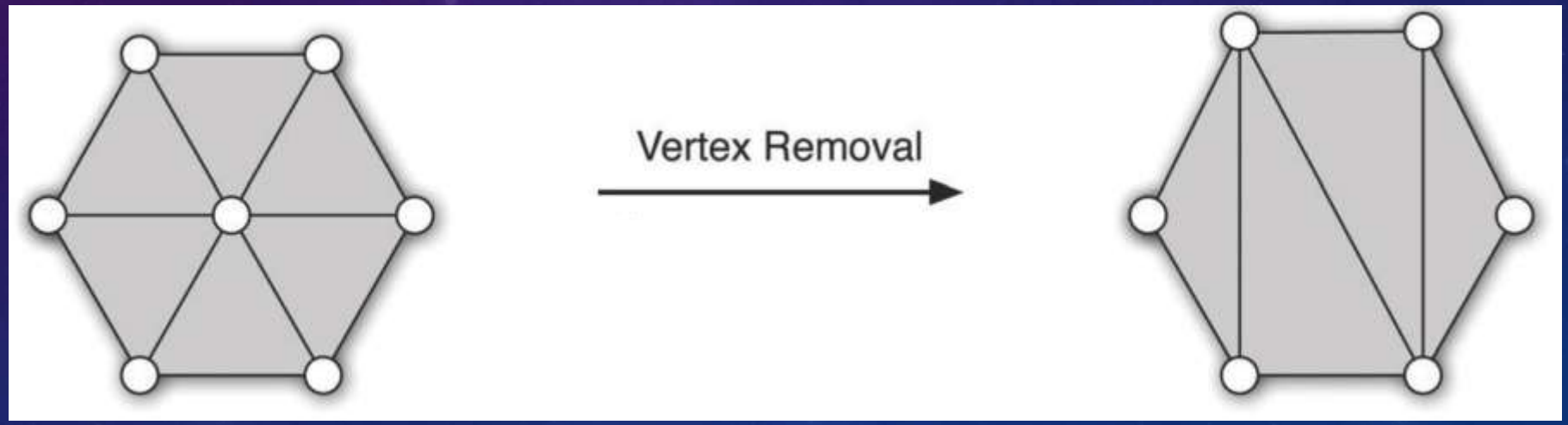
- Approximation error
- Other criteria (curvature, texture)

Local operations

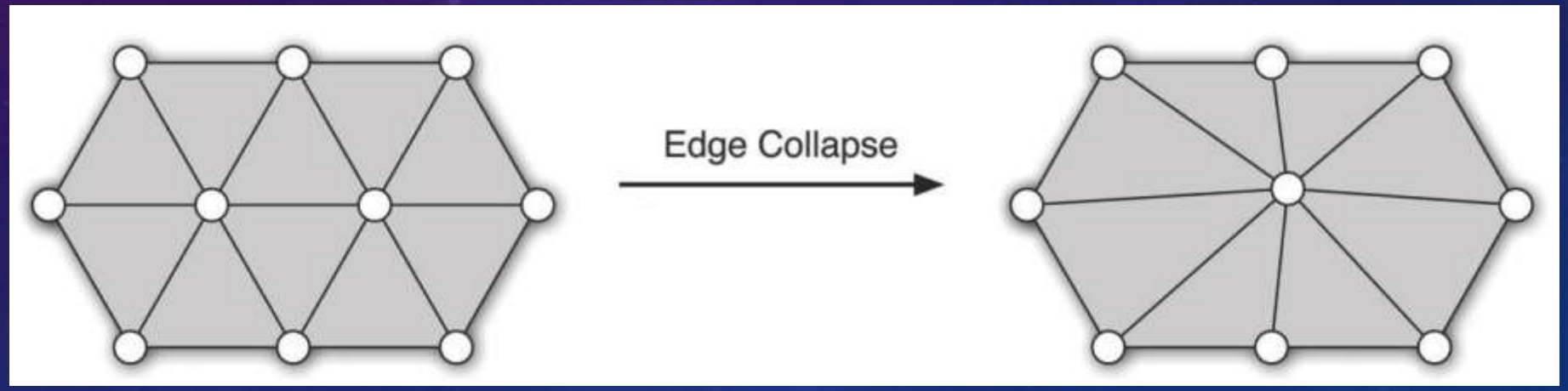
Topological operations

- Decimation (vertex removal)



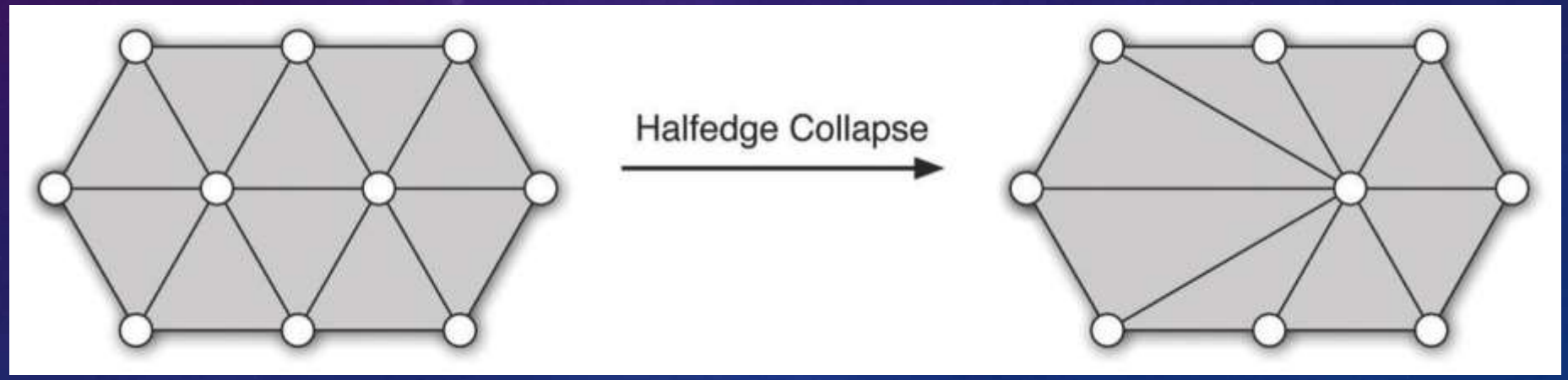
Topological operations

- Decimation (edge collapse)



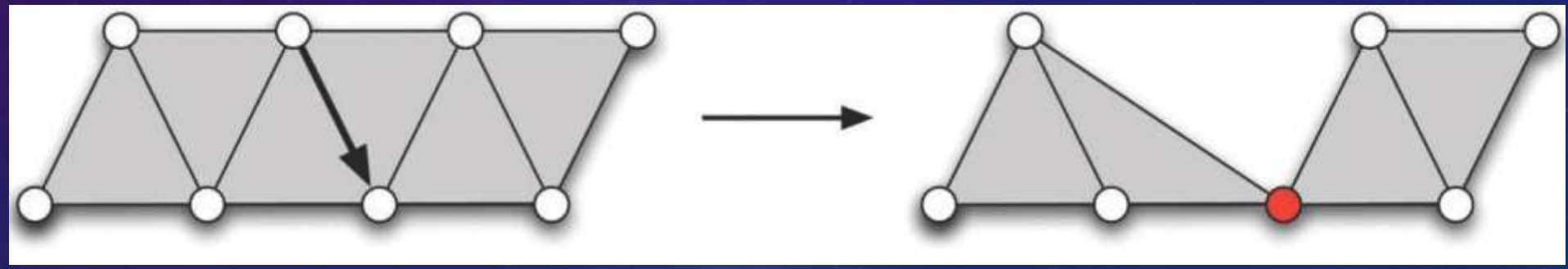
Topological operations

- Decimation (half-edge collapse)



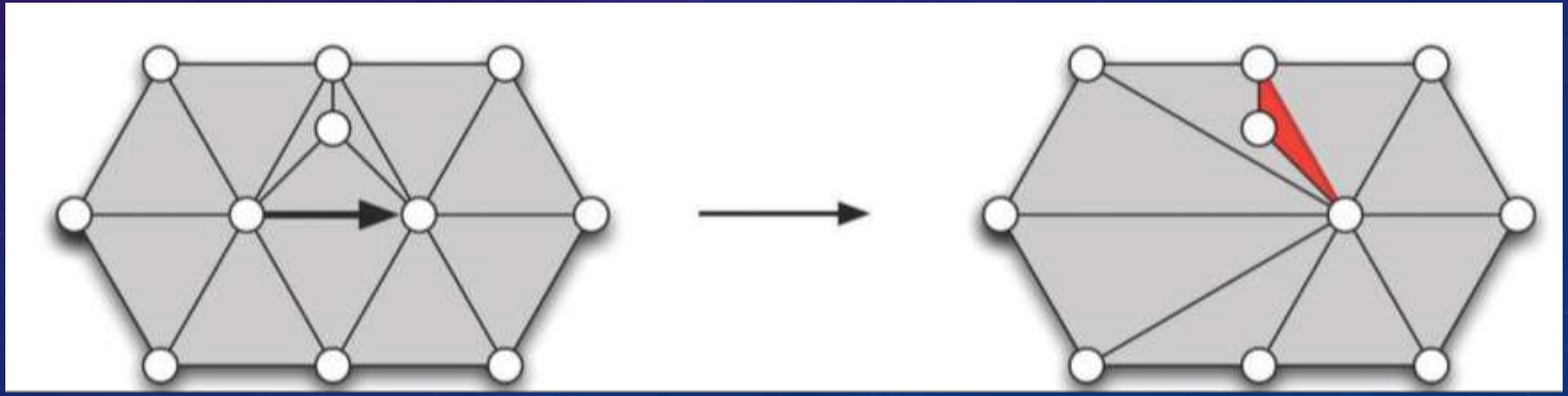
Illegal (half-)edge collapses

- If p and q are boundaries vertices, then edge (p,q) should be a boundary edge.



Illegal (half-)edge collapses

- If i and j are boundaries vertices, then edge (i, j) should be a boundary edge.
- For each k incident to both i and j, {i, j, k} should be the vertices of a triangle.



Quadric error metric

Quadric error metric (QEM)

- The squared distance of a point x from the plane P:

$$d(x,P)^2 = \left(n^T x - d\right)^2, d = n^T y$$

Denote $\bar{x} = (x, 1)$ and $\bar{n} = (n, -d)$, then

$$d(x,P)^2 = \left(\bar{n}^T \bar{x}\right)^2 = \bar{x}^T \bar{n}\, \bar{n}^T \bar{x} \triangleq \bar{x}^T Q_P \bar{x}$$

Quadric error metric (QEM)

- For vertex i, $Q_i \triangleq \sum_{ijk} Q_{ijk}$

$$d(x, Q_i)^2 = \bar{x}^T Q_i \bar{x} = \begin{pmatrix} x^T, 1 \end{pmatrix} Q_i \begin{pmatrix} x \\ 1 \end{pmatrix}$$

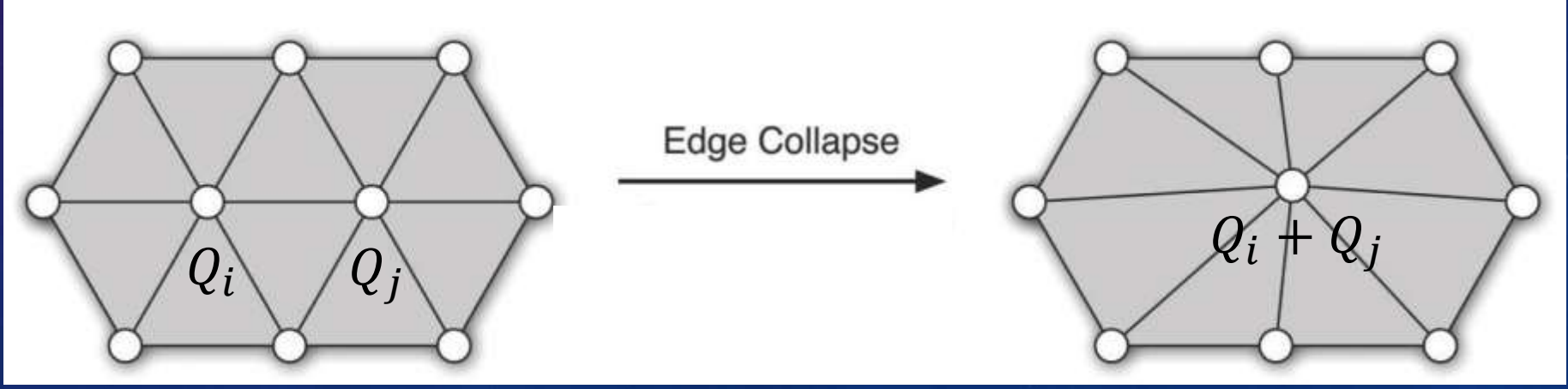
When $x = v_i$, $d(x, Q_i)^2 = 0$

Quadric error metric (QEM)

- When edge (i, j) collapses,

$$d^2 = d(x, Q_i)^2 + d\left(x, Q_j\right)^2 = \bar{x}^T (Q_i + Q_j)\bar{x}$$

- New position $x = \arg\min \bar{x}^T (Q_i + Q_j)\bar{x}$



QEM algorithm

Surface simplification using quadric error metrics
[Garland & Heckbert 1997]

- Input: a mesh
- Output: a simplified mesh

Initialization:

- Compute the matrices Q_i for each vertex i
- Compute the optimal contraction target v for each edge (i, j)

While $N_V > N \; and \; Cost_{min} < t$

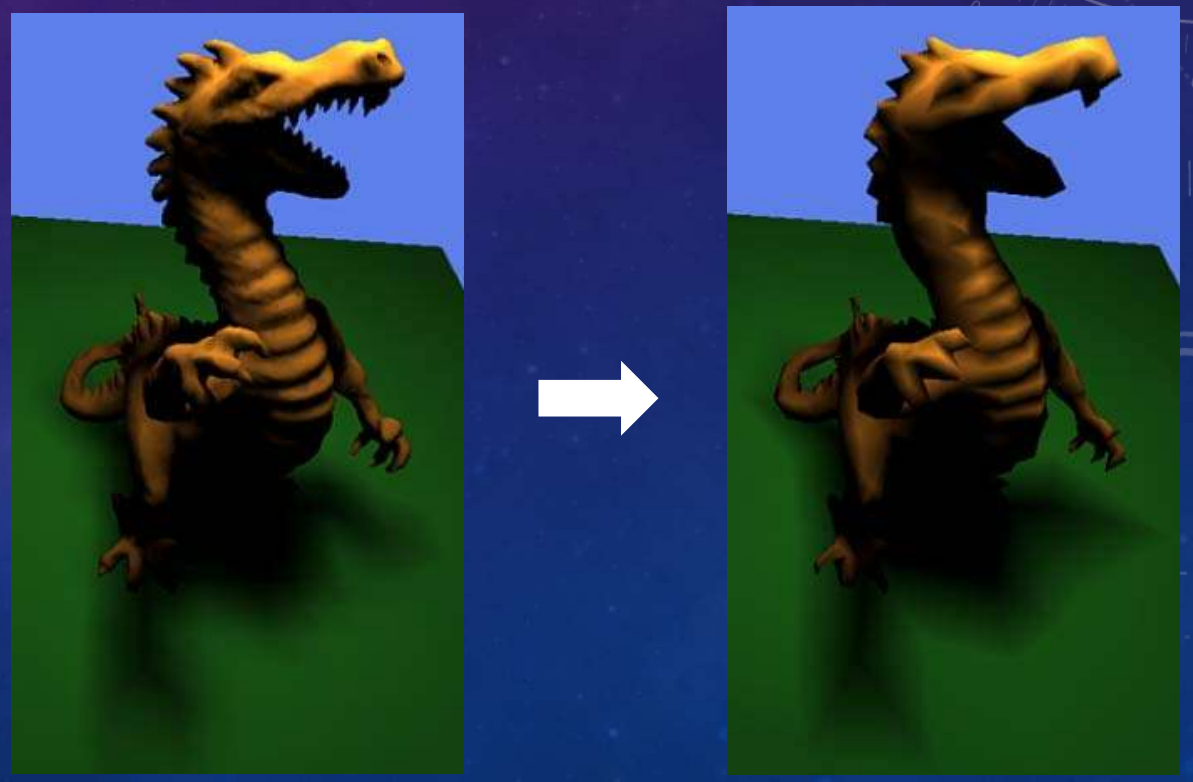
- The error $v^T(Q_i + Q_j)v$ becomes the cost of the edge (i, j)
- Place all the edges in a priority queue keyed on cost with minimum cost edge at the top.
- Remove the edge of the least cost from the heap , collapse this edge, and update the costs of all edges involving.

End

Extension

Vertex attributes Become added dimensions

- Color (x, y, z, r, g, b)
- Texture (x, y, z, u, v)
- Normal (x, y, z, n_x, n_y, n_z)
- …



Assignment requirements

- QEM algorithm
- Email: ID_name_homework#1.zip
  - Pdf : Input + parameter + output
  - Source code (no exe)
- Deadline: 2024.03.10, 23:59